Ontario

Ministry of Education

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GRADES 7, 8 MATHEMATICS, NUMERICAL METHODS

NOTES FOR TEACHERS

CONTENTS

(Notes for Grade 7 N1,N5 to N8 and Grade 8 N2 to N7 are under preparation and will be distributed later.)

The resource notes in this module are related to the Grades 7 and 8 Numerical Methods Strands for Intermediate Division Mathematics 1977, Draft Copy. They are intended for use by teachers and board curriculum committees as they plan the mathematics program for their schools.

Ministry of Education Ontario

GRADE 7 NUMERICAL METHODS

SECTION 2: NUMBER APPLICATIONS

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 6, 11
	Ed PJ Div:	Pages 61-74 From Counting to Calculation (a resource module)
PRESENT	Gr 7:	N 3; N 4; N 6; N 8; A 1c; A 2; G 5b
FUTURE	Gr 8:	N 1; N 2; N 3; N 4; N 5; N 6; N 7; A 1b-e; A 4; G 3b; G 4bc
	Gr 9 Gen:	N 1; N 2; N 3; N 4; N 5; N 6; N 7; A 1bc; A 2ad; G 1d; G 2; G 4; G 5b
	Gr 9 Adv:	N 1; N 2; N 3; N 4; N 5; N 6; A 1bc; A 2ad; G 1b; G 2; G 5bc
	Gr 10 Gen:	N 1; N 2; N 3; N 4; N 5; N 6; N 7; N 8; G 1b; G 2; G 3
	Gr 10 Adv:	N 1; N 2; N 3; N 4; A 1d; G 1; G 2; G 5

The Role of the Calculator in Developing this Section

The mini-calculator is used extensively by the general public for purposes ranging from everyday computations to scientific, engineering, business, and statistical applications.

Outside the school the decision to use the calculator is made by the student, and in some instances by the parents. Within the school, this decision is often made by the board, by the principal, or by the individual teacher, all of whom are concerned that students develop a reasonable knowledge of number facts and an ability to apply them in everyday computations.

At present, no one can predict the long-range effects of the calculator with assurance. The next decade will likely clarify the matter and even then the effects will vary from student to student, depending on the way the calculator is used in the classroom.

On the basis of present knowledge, however, it is safe to say that the calculator should not replace traditional experiences with mental arithmetic and simple pencil and paper calculations. It should complement these activities by eliminating the more tedious arithmetic. Any use of the calculator should also reinforce and extend the students' factual knowledge and their conceptual understanding related to numbers, number sense, and numeric problem-solving skills.

Students should be encouraged to learn from the calculator and to use it efficiently. They will still need to know the basic number facts and to use them independently of the calculator in simple situations and to make mental checks on the calculator's answers. The notes that follow will indicate some of the ways the calculator may be used in developing the program.

See also the notes for topic 8N 1f.

a) Place value

The notion of place value should be well established by this time through a variety of experiences in the K-6 program. Observation of the students' work should reveal any weaknesses in their understanding of place value and so suggest the kinds of additional experiences needed, probably on an individual basis.

The students should know the expanded form for writing a numeral, as suggested by the examples below.

$$5326 = 5 \times 1000 + 3 \times 100 + 2 \times 10 + 6 \times 1$$
$$352.67 = 3 \times 100 + 5 \times 10 + 2 \times 1 + 6 \times \frac{1}{10} + 7 \times \frac{1}{100}$$

They should also be able to identify for any numeral the units, tens, hundreds, and thousands digits, and when working with decimals, they should be able to identify the tenths, hundredths, and thousandths digits, and so on.

When dealing with topic N 5b, they should be able to express a numeral in expanded form involving powers of ten. For example: $3784 = 3 \times 10^3 + 7 \times 10^2 + 8 \times 10 + 4 \times 1$. Since they are not yet familiar with integral exponents, the students in general should not be expected to learn that $1 = 10^0$, $0.1 = 10^{-1}$, $0.01 = 10^{-2}$, and so on. This would seem to be more appropriate after Sections 8N 4, and 9Adv N 3 or 9Gen N 3.

The notion of place value might be discussed in relation to the display on a mini-calculator. A number such as 563 429 is entered into the calculator digit by digit, starting with the 5 and working to the right, as illustrated below.

Enter	Display
	0.
$\boxed{5}$	5.
$\boxed{6}$	56.
$\boxed{3}$	563.
$\boxed{4}$	5634.
$\boxed{2}$	56342.
$\boxed{9}$	563429.

The digit 5 successively takes a place value of units, tens, hundreds, thousands, ten thousands, hundred thousands. As each new digit is entered, its value is successively multiplied by 10: 5, $5 \times 10 = 50$, $50 \times 10 = 500$, $500 \times 10 = 5000$, $5000 \times 10 = 50\,000$, $50\,000 \times 10 = 500\,000$.

A similar discussion applies to each of the other digits 6, 3, 4, 2, and 9. However, when a decimal such as 739.256 is entered, the above discussion applies only to the digits to the left of the decimal point. The digits to the right of the decimal point maintain the same value as successive digits are entered, as illustrated below.

Enter	Display
	0.
$\boxed{7}$	7.
$\boxed{3}$	73.
$\boxed{9}$	739.
$\boxed{\cdot}$	739.
$\boxed{2}$	739.2
$\boxed{5}$	739.25
$\boxed{6}$	739.256

The print media frequently uses a combination of a numeral and a word to denote large numbers, as illustrated in the clippings below. This 'numeral-word' notation is closely related to scientific notation: 9 million = 9×10^6 (see 8N 5e, and 9Adv 3c or 9Gen 3f) and so deserves some attention.

Bicycle Sales Up to 9 Million Last Year

Americans bought more than nine million bicycles last year, according to estimates by the Bicycle Manufacturers Assn. of America — up from 6.2 million in 1967, and 7.3 million in 1975.

And industry surveys indicate continued sales increases — to 11 million by 1980 and 19 million by 1990.

The total number of bicycles in use today is estimated at 90 million.

14 The Toronto Sun, Thursday November 16, 1978

3 billion years left for us to have good time

EARTH'S LIFE SPAN PREDICTED BY PROF

GATLINBURG, Tenn. (UPI-Special) — "Come let us take a muster speedily: Doomsday is near; die all, die merrily."

We really don't have much to worry about in that respect. According to new data, the Earth won't become a "dead" planet for 3 billion more years.

That is, of course, if man doesn't destroy it first.

N. Mafi Toksoz, a geophysics professor at Massachusetts Institute of Technology, said his prediction on earth's life span is based on discoveries about the evolution and history of planets made through space exploration in the last decade.

He told the 16th annual briefing of the Council for the Advancement of Science Writing the most up-to-date data shows Earth's crust has changed composition significantly from when it was formed 4.6 billion years ago.

Numbers in this form may be used to provide a simple introduction to algebra. For example, in the article on bicycle sales the increase from 1967 to 1975 in bicycle sales was 7.3 M - 6.2 M = 1.1 M. The symbol M is used here to represent 1 000 000. This simple example could lead to further work with placeholders or variables.

Words such as million and billion become particularly important in problems such as: "Find the difference between 1.2 billion and 800 million".

b) <u>Operations with whole numbers and decimals; estimating answers; applications in real-life situations</u>

These topics are an extension of experiences of the K-6 program. Here the emphasis should be on consolidating skills and concepts and in applying them in new situations.

<u>Operations with whole numbers and decimals</u>

Calculations with pencil and paper should stress the accurate use of algorithms and will help teachers in identifying students who need individual help with fundamental principles. The calculations need not involve large numerals. The position of the decimal point in multiplication and division may need additional attention and practice.

It is helpful to link computation to circumstances related to a student's living habits. For example, division is made more meaningful in the context of a problem such as: "How many 35¢ comic books can I buy for $3.25?".

<u>Estimating answers</u>

Students should be encouraged to estimate answers and to assess how reasonable computed results are; they should reconsider problems where computed answers and expected answers substantially disagree. This skill is of particular value when working with a calculator.

Applications in real-life situations

Real-life circumstances can be included in a variety of ways. For example, a newspaper clipping such as the one shown can provide material for problems involving whole numbers and decimals. Questions such as the following might be asked.

2.5 Billion Pounds of Explosives Produced In U.S. in 9 Months

From January through September of 1977, about 2.5 billion pounds of explosives were manufactured in the U.S., according to the Bureau of Alcohol, Tobacco and Firearms.

During that same period, about 28,900 pounds of explosives and blasting agents, and 36,000 blasting caps were reported stolen.

And during those nine months, 25 deaths and 127 injuries were attributed to criminal bombings, and 24 deaths and 103 injuries resulted from accidental explosions.

"How many people died from explosions?"

"How many injuries were a result of explosions?"

"By what amount did thefts of blasting caps exceed those of explosives?"

"What is the average monthly explosive production?"

Including the clipping in the student exercise reinforces reading skills as well.

Sports activities such as football provide practical examples for numerical calculations. Total yards for, against, on a touchdown play, per player, or in penalties, lend excellent opportunity for operations with one and two digit numbers.

For many students, keeping a summary of their own income and expenses for several weeks will provide useful calculations with decimals.

Finally, just having a student recognize and record occasions where he/she has used some computation over a short period of time can be highly motivating.

c) <u>Practice in computation through games, puzzles, and activities</u>

Games, puzzles, and activities present an easy and interesting way to practice computation. Calculators can be used in many instances to shift the emphasis of the activity.

<u>Example 1</u> <u>Calculator Nim</u>

Two players alternate in attempting to reduce 21 to 0 by successive subtractions of 1, 2, or 3 from the subtotal. The person who makes the subtotal of 0 <u>loses</u>. For example:

Player A	21 → 19		15		11		6		3		0
Player B		16		13		8		4		1	

A has no choice but to remove the last 1, and so make the subtotal of 0; A loses.

Some students may discover the winning strategy for this game.

<u>Example 2</u> <u>Interceptor</u>

A rocket must destroy a Martian space ship 100 km above the earth. The rocket only has fuel for 86 km. By what factor must the

quantity of fuel be multiplied so that the rocket can reach the space craft? (Use multiplication only.) This problem can be explored in a number of ways, as illustrated.

Variation 1 Two-person, one calculator game

The players use the same calculator, and adjust the product calculated by the previous player.

Turn No.	Player A	Player B
1	86 x 1.5 = 129	129 x 0.8 = 103.2
2	103.2 x 0.98 = 101.136	101.136 x 0.99 = 100.12464
3	100.12 x 0.999 = 100.02451	100.02451 x 0.999 = 99.924485

↑ winner

Variation 2 Two-person, two calculator game

The players each use their own calculators and see who is closest to the target number after four turns.

Turn No.	Player A	Player B
1	86 x 1.3 = 111.80	86 x 1.2 = 103.2
2	111.80 x 0.95 = 106.21	103.2 x 0.93 = 95.976
3	106.21 x 0.96 = 101.9616	95.976 x 1.06 = 101.73456
4	101.9616 x 0.99 = 100.94198	101.73456 x 0.99 = 100.7121

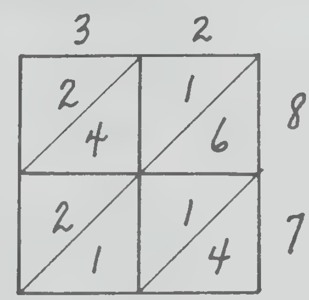
↑ winner

Variation 2 is the more instructive since each player can work out a strategy based upon his/her own results. Moreover, seeing the results of the other player may allow further refinement of guesses.

This game may also be played with division; however, in this case a distance such as 127 km would be appropriate; otherwise the problem becomes trivial.

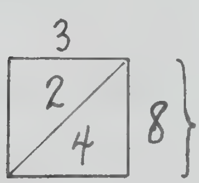
<u>Example 3</u>   <u>Grisley Grids</u>

The figure on the right is an example of a '<u>Grisley Grid</u>'.

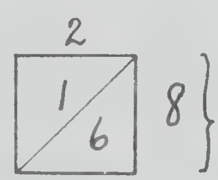


Each small square contains a two-digit numeral, with its tens digit above the diagonal and its units digit below. The factors of each numeral are written outside the large square at the top and to the right.

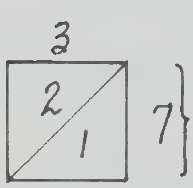
For example:



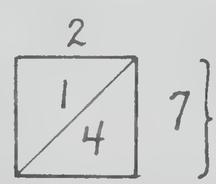
24 = 3 x 8



16 = 2 x 8

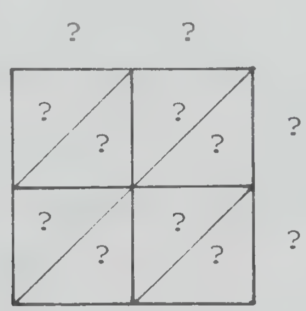


21 = 3 x 7

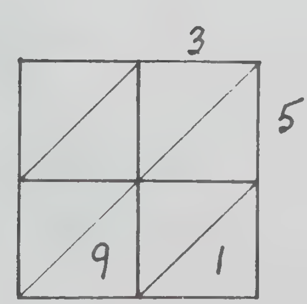


14 = 2 x 7

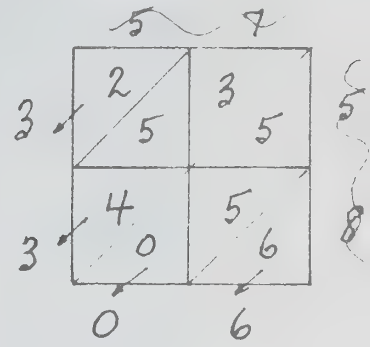
(i) Ask the students to make their own Grisley Grids.



(ii) Provide the students with grids that are partly completed. Ask them to complete the grids. For example:



(iii) Ask the students to find the value of Grisley Grids by:



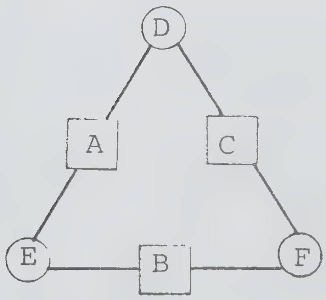
- adding the digits in the diagonal columns (as shown; do not add the factors; use the usual rules of column addition);
- entering the result 3306 below the diagonal columns, as shown;
- finding the sum of the digits $3 + 3 + 0 + 6 = 12$

The value of the grid is 12

Example 4 Addition-subtraction triangles

Triangles may be used to practise addition and subtraction.

Properties

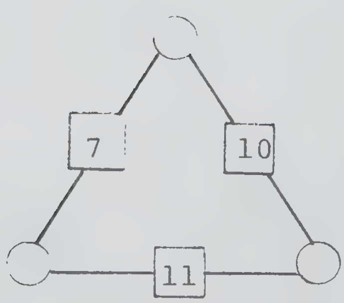


$A = D + E$

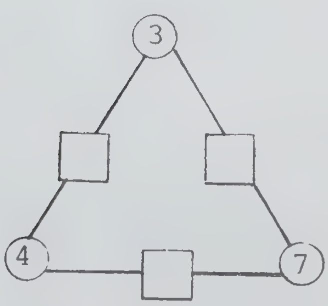
$C = D + F$

$B = E + F$

Sample Problem

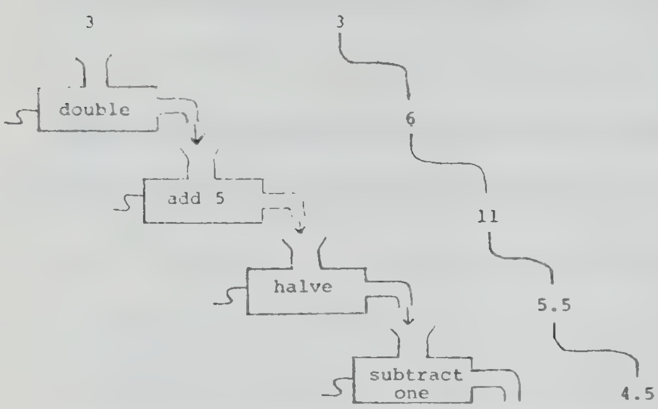


Solution



Example 5

Simple ideas involving flow charts and functions can be introduced in an informal way by using function machines.



Now change the order of the machines to see the effect on the result.

Which order produces the largest result? Which order the smallest?

Example 6

Teachers should have on hand some simple cross-number puzzles. These are self checking, motivating, and non-threatening.

Example 7

Unusual problems, particularly those involving measurement and estimation provide computational practice. Some of them might have a 'zany' appeal; for example:

i) How many ice cubes would fill your freezer?

ii) How many brush or comb strokes does your hair receive every year?

iii) How many coffee beans — or tea bags — would be needed to equal your mass?

iv) How many cereal boxes would fit inside your classroom? How long would it take the class to eat the cereal?

d) <u>Rounding; applications in real-life situations</u>

Rounding procedures are usually used when estimating, in order to make the estimates more reliable. Students should realize that the extent to which the rounding process is applied depends on the use to be made of the rounded figure. For example, it is useful to round the National Debt to the nearest million dollars, but, for most people, it would be senseless to round a personal debt other than to the nearest ten or hundred dollars.

The process of rounding is important for the students to understand. There are many different processes used when rounding a numeral; the process depends on the person or institution doing the rounding and the use to be made of the rounded numerals.

Traditionally, if the digit 'd', when adjusted, is followed by a digit of:

i) 5 or larger: then d is increased by one;

ii) 4 or less: then d remains the same.

For example:

i) 3.45, 3.46, 3.47, 3.48, and 3.49 are each rounded to 3.5;

ii) 3.44, 3.43, 3.42, 3.41, and 3.40 are each rounded to 3.4.

A slightly more sophisticated process when the digit that follows d is 5 is gaining popularity, as illustrated by the following examples.

3.4500 rounds to 3.4 (because 4 is even)

3.7500 rounds to 3.8 (because 7 is odd)

3.4501 rounds to 3.5 (because $0.4501 > 0.4500$)

The above processes are not commonly used in grocery stores, where a decimal value is usually rounded upward. For example, 23.7¢ becomes 24¢ (the same as above), but 23.2¢ also becomes 24¢. Many insurance companies and banks now round to the nearest even digit; for example, 23.2¢ becomes 24¢ and 22.8¢ becomes 22¢.

Truncation is a process in which all digits after a specified digit are dropped. For example 3.666 666 666 ... when truncated to 8 digits becomes 3.666 666 6.

The display in most four-function calculators is truncated to 8 digits; for example, $\boxed{5}$ $\boxed{\div}$ $\boxed{3}$ produces a display of 1.6666666. (If the machine had rounded, the display would have been 1.6666667.) This truncation feature sometimes leads to small inconsistencies. For example, $\boxed{2}$ $\boxed{\div}$ $\boxed{3}$ $\boxed{x}$ $\boxed{3}$ produces a display of 1.9999998, and not 2. This occurs because $\boxed{2}$ $\boxed{\div}$ $\boxed{3}$ produces 0.6666666 and when this is multiplied by 3 the result is 1.9999998. If the machine had rounded, then $\boxed{2}$ $\boxed{\div}$ $\boxed{3}$ would give 0.6666667 and when this is multiplied by 3 the result is 2.0000001.

When truncated to two digits, the numerals 32 405 , 32 999, and 32 003 each become 32 000.

The usefulness of rounding can be illustrated easily by an everyday example, such as counting the number of seats in the auditorium. Count the number of seats in a row, and the number of rows. Suppose there are 28 seats per row and 41 rows. Round each number to one digit accuracy, that is 30 and 40; then calculate the number of seats as 30 x 40 = 1200. This is a good estimate of the number of seats; the actual number is 1176.

Consider the effect of rounding and truncation on the following product: 2.25 x 2.35

<u>Rounding up</u>: 2.25 x 2.35 becomes 2.3 x 2.4 = 5.52

<u>Rounding to nearest even digit</u>: 2.25 x 2.35 becomes 2.2 x 2.4 = 5.28.

<u>Truncating</u>: 2.25 x 2.35 becomes 2.2 x 2.3 = 5.06.

The actual product is 5.2825. A discussion of the implications of these procedures in chain calculations is appropriate at this level.

e) <u>Multiplication and division with powers of ten</u>

The calculator may be used effectively in this topic. The students can see exactly what happens as a number is multiplied or divided by ten. Emphasize that there is no change in the digits or in their order when a numeral is multiplied or divided by a power of ten. In most calculators, the digits move sometimes, the decimal point on other occasions, depending upon the stage of the calculation;

for example:

Operation	Display
	1500.
$\div$ 10 =	150.
$\div$ 10 =	15.
$\div$ 10 =	1.5
$\div$ 10 =	0.15
$\div$ 10 =	0.015

f) Properties of arithmetic operations, and the roles of 0 and 1 identified in b) and c)

Familiarity with the properties of the operations is not only useful to computation, but helps a student to organize arithmetic concepts more easily. While the general form of these properties is not encouraged at this grade, recognizing and using them in numerical cases is appropriate.

Numerical		In General
i) $(3 + 2) + 5$ $= 5 + 5$ $= 10$	$3 + (2 + 5)$ $= 3 + 7$ $= 10$	
$\therefore (3 + 2) + 5 = 3 + (2 + 5)$		$(a + b) + c = a + (b + c)$
ii) $(3 \times 2) \times 5$ $= 6 \times 5$ $= 30$	$3 \times (2 \times 5)$ $= 3 \times 10$ $= 30$	
$\therefore (3 \times 2) \times 5 = 3 \times (2 \times 5)$		$(a \times b) \times c = a \times (b \times c)$
iii) $6 + 5$ $= 11$	$5 + 6$ $= 11$	
$\therefore 6 + 5 = 5 + 6$		$a + b = b + a$
iv) 6×5 $= 30$	5×6 $= 30$	
$\therefore 6 \times 5 = 5 \times 6$		$ab = ba$
v) $2(3 + 8)$ $= 2(11)$ $= 22$	$2 \times 3 + 2 \times 8$ $= 6 + 16$ $= 22$	
$\therefore 2(3 + 8) = 2 \times 3 + 2 \times 8$		$a(b + c) = ab + ac$
vi) $7 + 0 = 7$		$a + 0 = a$
vii) $7 \times 1 = 7$		$a \times 1 = a$

Counterparts of the above properties do not hold in most cases for the operations of subtraction and division. This may be shown during the discussion of section 7N 6 or 8N 4.

	Numerical		In General
i)	$(3 - 2) - 5$ $= 1 - 5$ $= -4$	$3 - (2 - 5)$ $= 3 - (-3)$ $= 6$	
$\therefore$	$(3 - 2) - 5 \neq 3 - (2 - 5)$		$(a - b) - c \neq a - (b - c)$
ii)	$(3 \div 2) \div 5$ $= 1.5 \div 5$ $= 0.3$	$3 \div (2 \div 5)$ $= 3 \div (0.4)$ $= 7.5$	
$\therefore$	$(3 \div 2) \div 5 \neq 3 \div (2 \div 5)$		$(a \div b) \div c \neq a \div (b \div c)$
iii)	$6 - 5$ $= 1$	$5 - 6$ $= -1$	
$\therefore$	$6 - 5 \neq 5 - 6$		$a - b \neq b - a$
iv)	$6 \div 5$ $= 1.2$	$5 \div 6$ $= 0.83333333...$	
$\therefore$	$6 \div 5 \neq 5 \div 6$		$a \div b \neq b \div a$
v)	$2(3 - 8)$ $= 2(-5)$ $= -10$	$2 \times 3 - 2 \times 8$ $= 6 - 16$ $= -10$	
$\therefore$	$2(3 - 8) = 2 \times 3 - 2 \times 8$		$a(b - c) = ab - ac$

vi)	$2 \div (3 - 8)$ $= 2 \div (-5)$ $= -0.4$	$2 \div 3 - 2 \div 8$ $= 0.6666666 - 0.25$ $= 0.4166666$	
	$\therefore\ 2 \div (3 - 8) \neq 2 \div 3 - 2 \div 8$	$a \div (b - c) \neq (a \div b) - (a \div c)$
vii)	$7 - 0 = 7$	$a - 0 = a$
viii)	$7 \div 0$ is undefined	$a \div 0$ is undefined $(a \neq 0)$

g) Use of these properties in simplifying numerical expressions

Addition and subtraction problems may be simplified by grouping numbers which have a sum of 0.

Example

$$\begin{aligned} & 7 - 3 + 8 - 1 - 6 \\ &= (7 - 1 - 6) + (8 - 3) \\ &= 0 + 5 \\ &= 5 \end{aligned}$$

In section 7N 4 the basic property that permits the addition of fractions is the identity for multiplication along with the concept of equivalent fractions. For example:

$$\begin{aligned} \frac{1}{3} + \frac{2}{7} &= \frac{1}{3} \times 1 + \frac{2}{7} \times 1 \\ &= \frac{1}{3} \times \frac{7}{7} + \frac{2}{7} \times \frac{3}{3} \\ &= \frac{7}{21} + \frac{6}{21} \\ &= \frac{13}{21} \end{aligned}$$

Students sometimes have difficulty in choosing a useful equivalent for 1. In the above example, why is 1 replaced first by $\frac{7}{7}$, then by $\frac{3}{3}$?

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GRADE 7 NUMERICAL METHODS

SECTION 3: PERCENT

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 6, 11
	Ed PJ Div:	Pages 67-71
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FUTURE	Gr 8:	N 1; N 2; N 3; N 5; N 6; A 1ad; A 3; G 4c
	Gr 9 Gen:	N 1; N 2; N 4; N 5; N 6; A 5; G 4c
	Gr 9 Adv:	N 1; N 2; N 4; N 5; A 5
	Gr 10 Gen:	N 1; N 2; N 3; N 5; N 6
	Gr 10 Adv:	N 3; A 3

a) Percents as hundredths

At this level the approach to percent should emphasize that percent means 'per hundred' and the % symbol is simply another way of writing $\frac{1}{100}$. Alternately, the % symbol may be replaced by 0.01; thus 5% means 5 x 0.01 or 0.05.

Computation with percents may be done in fractional or decimal form. For example, 7% of \$9.00 = $\frac{7}{100}$ x \$9.00 = \$0.63 or alternately 7% of \$9.00 = 0.07 x \$9.00 = \$0.63.

Newspapers, shopping catalogues, advertisements (especially those for supermarket sales), are excellent sources of everyday problems involving percentage savings. Sales tax should be discussed and calculated.

The calculator can be used effectively in dealing with realistic problems, once a general understanding is reached about the concept; for example: 15% of \$54.60 may be determined by entering
$\boxed{5}$ $\boxed{4}$ $\boxed{.}$ $\boxed{6}$.
Note that the commutative property of multiplication must be used when entering percentage data in most calculators. Entering $\boxed{1}$ $\boxed{5}$ $\boxed{\%}$ $\boxed{\times}$ $\boxed{5}$ $\boxed{4}$ $\boxed{.}$ $\boxed{6}$ $\boxed{0}$ yields 815 in the display. In this form the % does not register. Also observe when entering $\boxed{5}$ $\boxed{4}$ $\boxed{.}$ $\boxed{6}$ $\boxed{0}$ $\boxed{\times}$ $\boxed{1}$ $\boxed{5}$ $\boxed{\%}$ that the result 8.19 appears when the $\boxed{\%}$ key is pressed. If the $\boxed{=}$ key is depressed after the $\boxed{\%}$ key, the display reads 447.174 (the constant operation of "multiply by 54.60" has been programmed in the machine and the $\boxed{=}$ key activates this operation).

b) Conversion of percents to decimals and decimals to percents

Students will need practice in converting percents to decimals. In some cases they may place the decimal point incorrectly; for example:

150% may appear as 0.15 , instead of 1.5;

2.5% may appear as 0.25 , instead of 0.025;

12½% may appear as 0.0125, instead of 0.125.

It is best to use the actual division process until the students can convert to the decimal equivalent accurately on sight; for example:

$$12\tfrac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{12.5}{100} = 0.125$$

$$150\% = \frac{150}{100} = 1.5$$

The reverse process will identify the digit that occupies the hundredths place.

$$0.015 = \frac{1.5}{100} = 1.5 \times \frac{1}{100} = 1.5\%$$

$$1.255 = \frac{125.5}{100} = 125.5 \times \frac{1}{100} = 125.5\%$$

$$0.005 = \frac{0.5}{100} = 0.5 \times \frac{1}{100} = 0.5\%$$

Again it will be found, after a little practice, that most students will be able to make the conversion mentally.

Alternately, the principle of the identity for multiplication might be used. For example:

$$0.015 = 0.015 \times \frac{100}{100} = \frac{1.5}{100} = 1.5 \times \frac{1}{100} = 1.5\%$$

$$1.255 = 1.255 \times \frac{100}{100} = \frac{125.5}{100} = 125.5 \times \frac{1}{100} = 125.5\%$$

$$0.005 = 0.005 \times \frac{100}{100} = \frac{0.5}{100} = 0.5 \times \frac{1}{100} = 0.5\%$$

Once the principle is established and understood, the first and third steps can be done mentally; then the first, second, and third steps.

c) <u>Application to sales tax, simple interest, discount</u>

After a number of experiences in calculating sales tax, students might prepare a sales-tax table, stating the tax for sales up to $10. This will involve rounding procedures.

Some practice with the sales-tax table would be useful, especially when the purchase price is greater than $10.00. Here a mix of calculations involving work with powers of ten, together with addition of several values from the table may be required. For example, the tax on a sale of $125.48 would be found as follows:

0.70 x 10	+	0.70 x 2	+	0.38	(where $0.70 is the tax on $10.00 from the table)
(tax on $100)	+	(tax on $20)	+	(tax on $5.48 from the table)	
= 7.00	+	1.40	+	0.38	
= 8.78					

Simple interest can be taught in relation to students' savings accounts, allowance, or job incomes (paper route or lawn mowing). Most newspapers carry advertisements concerning 'sales' and 'specials'. This material could be used as a basis for work on discounts expressed as percentages.

d) Applications to gain or loss

This topic may be introduced by increasing a number by 10% and then decreasing the result by 10%. For example:

$$\$100 + 10\% \text{ of } \$100 = \$110$$

$$\$110 - 10\% \text{ of } \$110 = \$\ 99$$

Where did the $1.00 go? To students, this is a rather surprising result. Have them check this, and similar examples, with a calculator. For example:

Enter	Display
$\boxed{1}$ $\boxed{5}$ $\boxed{5}$	155.
$\boxed{+}$	155.
$\boxed{7}$ $\boxed{\%}$	10.85
$\boxed{=}$	165.85
$\boxed{-}$	165.85
$\boxed{7}$ $\boxed{\%}$	11.6095
$\boxed{=}$	154.42405

It should be noted that a percent of a number can be both calculated and added to (or subtracted from) the number directly on the calculator. See the example above. Steps 2 and 3 (7% of 155 is calculated), step 4 (added to 155), steps 5 and 6 (7% of 165.85 is calculated), step 7 (subtracted from 165.85).

Some students may be interested in the percentage gain (or loss) that occurs when a product is sold by a merchant. This may be calculated in two ways: as a percent of the cost price, or as a percent of the selling price.

For example:

$$\frac{(\text{selling price - cost price})}{\text{cost price}} \times 100\% = \text{percent gain based on cost price}$$

and

$$\frac{(\text{selling price - cost price})}{\text{selling price}} \times 100\% = \text{percent gain based on selling price}$$

Which method of calculating produces the largest 'percent gain'?

Students can be encouraged to find their own examples in the local newspaper. The use of calculators will allow for more realistic questions. (Note that a sale price can be considered a loss that the store assumes on the regular price.)

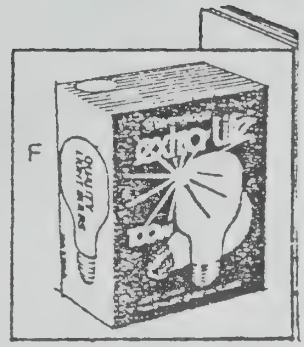
Example 1

Check these prices. Find (a) the price of the $11.95 sweatshirt less 16%, and (b) the price of the $7.49 sweatshirt less 16%.

Example 2

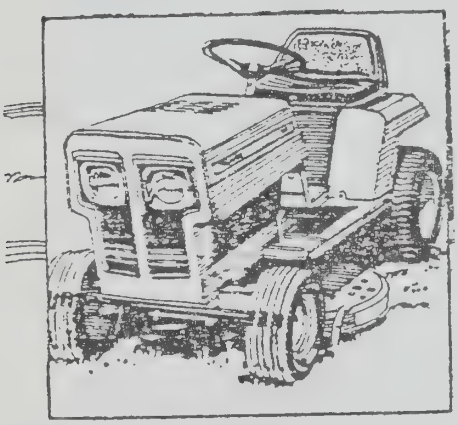
Save 17%-20%
Pkg. of 2 light bulbs
1.19
Pkg. Reg. $1.49

f-40, 60 or 100-w. 34R 016 509/10/11.
Pkg. of 2 Bug-a-way bulbs. 60-w.
34R 016 529. Reg. $1.69 Now $1.39



What percent is saved on the $1.19 package of light bulbs? on the $1.39 package?

Example 3



Save $80

8-hp lawn tractor
with 36" rotary mower

919.98 Reg. $999.98

What percent is the discount?

Example 4

SYDENHAM — A garbage feud which has simmered for nearly two years between Portland Township officials and area cottagers has intensified with the enforcement of a tough new anti - dumping bylaw.

The bylaw, passed Aug. 8, increases the fine for throwing garbage on any public or municipal property in Portland Township to $150 from $50.

What percent increase is there in the fine?

More student oriented examples would involve percent gains in height, mass, term marks, and so on.

Ministry of Education Ontario

GRADE 7 NUMERICAL METHODS

SECTION 4: FRACTIONS, RATIOS

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 6, 11
	Ed PJ Div:	Pages 61-74
PRESENT	Gr 7:	N lac; N 2; N 3ab; N 5a; A lc; A 2; G lcd; G 3eg; G 5; G 6e
FUTURE	Gr 8:	N ladf; N 2; N 3; N 5; N 6; N 7; A 1; A 3bcd; A 4; G 3ab; G 4
	Gr 9 Gen:	N 1; N 2; N 3c; N 4; N 5; N 6; A 1 A 3bcd; A 4; G 3ab; G 4
	Gr 9 Adv:	N 1; N 2; N 3d; N 4; N 5; A 1; A 2; A 4; A 5; G lb; G 2; G 4
	Gr 10 Gen:	N 1; N 2; N 3; N 4c; N 5; N 7; A 2 G 1; G 2; G 3
	Gr 10 Adv:	N 2; N 3; A 1; A 3; G 1; G 2; G 5

The Place of Fractions and Decimals in the Curriculum

Canada's adoption of the metric system raises significant questions about the place of fractions and decimals in the mathematics program and the emphasis that should be given to them.

Certainly decimals will be used extensively in practical mathematics, particularly in measurement. There is no doubt that, in preparing for the future, our students will need a complete understanding of decimals and must be comfortable when working with them. They will need facility in approximating, estimating, and rounding of numbers, and in knowing how to organize the solution of problems involving arithmetic operations. They must be able to carry out the operations, with pencil and paper in simple cases and with calculators in others, and to test the reasonableness of the results.

What then should be the place of fractions? Most teachers will agree that an understanding of fractions and the ability to operate with them in simple cases will continue to be significant in the foreseeable future. Notions related to simple fractions, such as one-half and two-thirds, will continue to be part of our everyday experiences. Notions of $\frac{1}{10}$, $\frac{1}{100}$, and so on are needed when introducing the meaning of the decimals 0.1 and 0.01. Fractions play an important role in computations associated with ratios. Fractions such as $\frac{1}{3}$ and $\frac{2}{7}$ are easier to understand and operate with than their non-terminating recurring

decimal equivalent forms 0.3 and 0.285714. Experiences with arithmetic fractions are considered prerequisite in algebra to the understanding of rational numbers and of the concepts of rational expressions and operations with them.

Concepts of fractions and decimals, and operations with them, are introduced together in the K-6 program. See The Formative Years, pages 6 and 11. An understanding of the meaning of the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, then $\frac{1}{10}$ and $\frac{1}{100}$ would seem to precede the meaning of the decimals 0.1 and 0.01 and of place value of the tenths and hundredths digits in a number such as 0.72. These ideas should have been well established in the K-6 program. The order in which a deeper understanding of fractions and decimals should be developed is a matter of personal choice. Traditionally, the study of fractions and operations with them have preceded the study of decimals. Today, there is strong opinion that many aspects of the concept of decimals and operations with them should precede the corresponding work with fractions. In the program for Grades 7 and 8, the fractions should be simple and easily related to practical examples. The students' experiences should establish the principles that are to be carried forward into algebra. There is little need for the students to practise the operational skills in complex cases; heavy arithmetic computation in most practical situations will be done with a calculator in the future.

a) Concept of a fraction

Balance of the Concrete, Semi-Concrete, and Abstract Stages

Students need many years to develop an understanding of fractions and of the skills of operating with them. Fractions have many

interpretations and subtle shades of meaning. Matching a suitable interpretation to a given situation is not always easy.

The numerical symbols for fractions such as $\frac{2}{3}$ or $\frac{3}{4}$, are abstract and have little meaning by themselves. In the earlier grades, these symbols have been introduced through the use of concrete materials (real-world objects and sets of objects that can be separated into parts) and of semi-concrete representations of them (pictures and diagrams; that is, models). Words such as 'one-half' and 'two-thirds' were used to describe parts of objects, pictures, and diagrams, and the abstract symbols $\frac{1}{2}$ and $\frac{2}{3}$ were introduced to represent them. This link between the fractional symbols and the concrete and semi-concrete situations should be maintained in the Intermediate Division, in order that understanding may be consolidated.

The stages of development in any class will likely range from concrete to semi-concrete to the abstract stage.

CONCRETE STAGE	SEMI-CONCRETE STAGE	ABSTRACT STAGE
real-world objects	pictures, diagrams	symbols, operations

The challenge for the teacher will be to identify the activities that can best help each student grow in understanding.

For students at the concrete or semi-concrete stage, both the concepts and operations should be revealed through the use of real objects or at the very least, pictures and diagrams that represent these objects. Examples of these will be discussed in the notes that follow.

Other students will be comfortable working with arithmetic skills using only numerical symbols. From time to time, however, they should be encouraged to make diagrams to illustrate the concepts and operations and to check their understanding. There is little value in practising the skills, if the students are unable to relate them to real situations and to check the reasonableness of their results.

The three stages of development are described below.

<u>Concrete Stage</u>

Students investigate concepts and operations using real objects. For example, they may use:

- an apple or other object that can be easily cut into pieces;
- Cuisenaire rods for comparing lengths;
- Centicubes for building objects that in turn can be divided into parts;
- paper for paper folding;
- geoboards;
- measuring devices such as a ruler, protractor, measuring cups or spoons;
- Tangram pieces;
- string;
- polyominoes;
- games that involve real objects to explore fractions.

<u>Semi-Concrete Stage</u>

Students investigate fraction concepts by using pictures or diagrams. For example, they may use:

- dot paper to make diagrams to represent geoboard-figures and their parts;
- grid paper or dot paper to construct squares, rectangles, and other figures that can be divided into parts;
- circles, divided into halves, quarters, or other parts;
- pictures of sets of things, in which the subsets can be 'circled' as parts of the whole set;
- number lines in which the positions of points are labelled with fractions;
- pictures of real-world objects that are often shared, such as a chocolate bar or pie;
- lines of symmetry to divide figures into fractions such as halves, quarters, and eighths;
- polar grid paper to show parts of circles;
- circle graphs to show percentage distribution of data;
- games that involve diagrams and other figures to explore fractions.

<u>Abstract Stage</u>

Students use numerical symbols, such as $\frac{2}{3}$, to represent fractional values and perform arithmetic operations with these symbols, according to established rules and without reference to concrete or semi-concrete representations. For example,

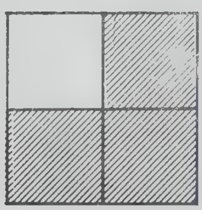
- reduce $\frac{8}{12}$ to lowest terms;
- simplify $\frac{2}{3} + \frac{4}{3} + \frac{1}{3}$;
- express $1\frac{3}{4}$ as a fraction;
- simplify $\frac{2}{3} + \frac{3}{4}$;
- simplify $\frac{2}{5} \times \frac{3}{4}$;
- simplify $\frac{1}{2} \div \frac{3}{4}$.

<u>Ways in which Fractions are Applied</u>

The following examples illustrate a number of circumstances in which fractions are used:

i) <u>to describe part of a whole object or figure</u>

<u>Example</u>

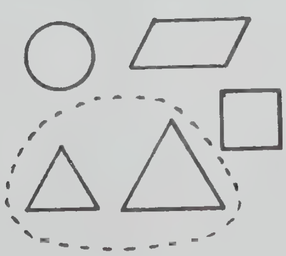


What part of the square is shaded?

Ans. $\frac{3}{4}$

ii) <u>to describe part of a set</u>

<u>Example</u>



What part of the set are triangles?

Ans. $\frac{2}{5}$

iii) to name points on a number line

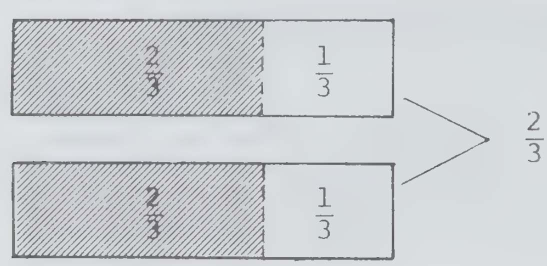
Example

What are the names of the circled points?

Ans. $\frac{3}{4}$ and $\frac{5}{4}$

iv) to describe each part when something is shared equally among n people

Example Two Chocolate Bars



Share the bars among three people. What part does each person get?

Ans. $\frac{2}{3}$ of a bar

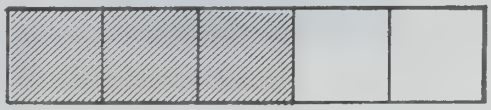
This example is essentially a division problem. Sharing t things among n people, each person gets $t \div n$ things; i.e. $\frac{t}{n}$ things.

Three Ways of Interpreting a Fraction

A fraction, such as $\frac{3}{5}$, may be interpreted in three different ways: as $\frac{3}{5}$ of 1, as $3 \times \frac{1}{5}$, and as $\frac{1}{5}$ of 3. These are illustrated by the practical situations below.

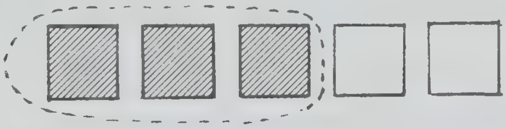
i) $\frac{3}{5}$ may mean $\frac{3}{5}$ x 1; i.e. $\frac{3}{5}$ of the 'whole'

(a) The diagram represents a chocolate bar with five 'squares'.



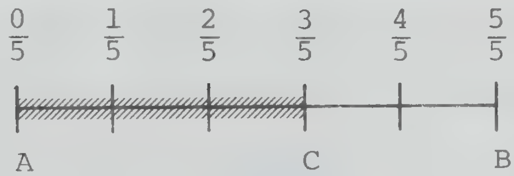
Myra takes three 'squares'. She has $\frac{3}{5}$ of the bar.

(b) The diagram represents the five records of an album.



Myra takes three of them. She has $\frac{3}{5}$ of the album.

(c) The diagram shows a segment AB, divided into five equal segments.

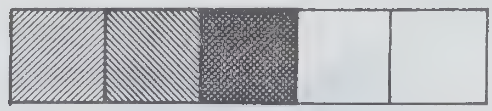


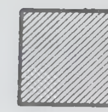
AC is $\frac{3}{5}$ of the length of AB.

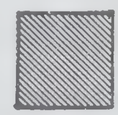
In these examples, the use of $\frac{3}{5}$ is closely related to the ratio concept; the shaded part is $\frac{3}{5}$ of the whole. The ratio of AC to AB is $\frac{3}{5}$:1 or 3:5.

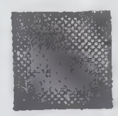
ii) $\frac{3}{5}$ may mean $3(\frac{1}{5})$; i.e. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

(a) The diagram represents a chocolate bar.



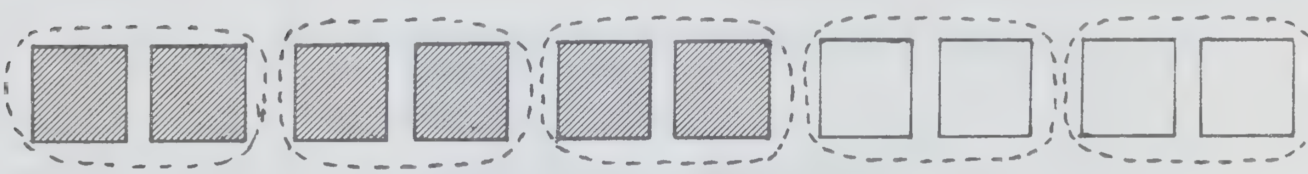
Myra takes the first square or $\frac{1}{5}$ of the bar. 

Allison takes the second square, 

and Jim the third. 

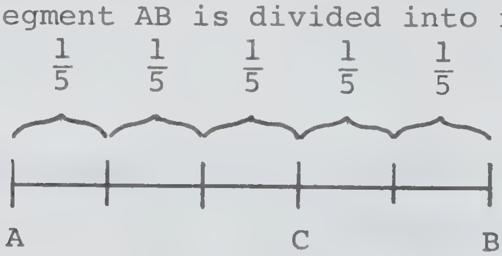
Altogether they took $(\frac{1}{5} + \frac{1}{5} + \frac{1}{5})$ or $3(\frac{1}{5})$ of the bar.

(b) The diagram represents the ten records of an album.



Myra, Allison, and Jim each take two records (the shaded records). Each took $\frac{1}{5}$ of the album. Altogether they took $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ or $3(\frac{1}{5})$ of the album.

(c) The segment AB is divided into fifths.

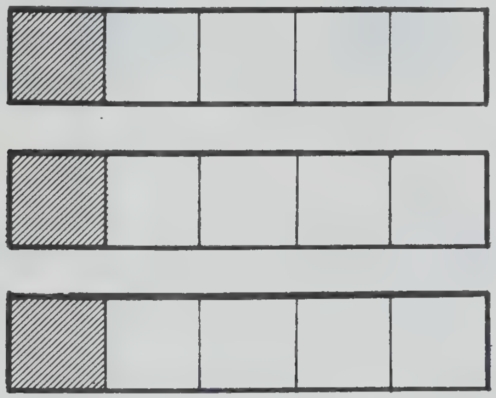


Segment AC is $\frac{1}{5}$ of AB + $\frac{1}{5}$ of AB + $\frac{1}{5}$ of AB or $3(\frac{1}{5})$ of AB.

In these examples $\frac{3}{5}$ is being used to represent the sum $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ or as $3(\frac{1}{5})$. The situations are algebraic in nature, and similar to $a + a + a = 3a$.

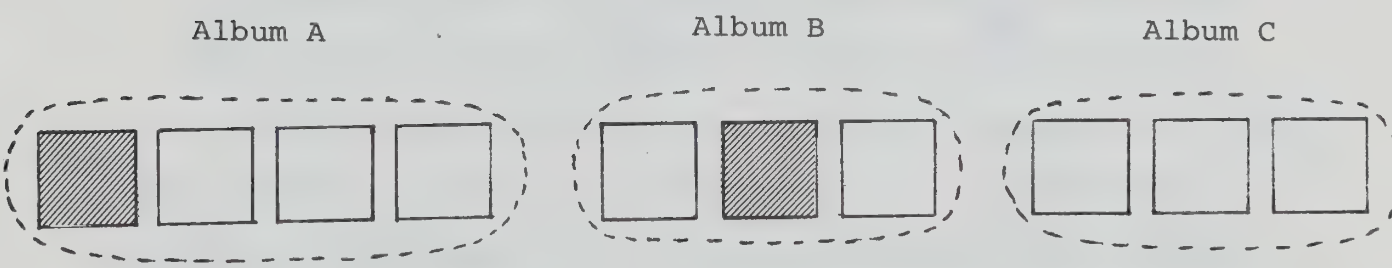
iii) $\frac{3}{5}$ may mean $\frac{1}{5}$ of 3

(a) The diagram represents three chocolate bars.



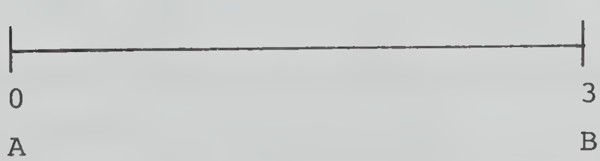
Rodney takes one square from each of the three bars; he has $\frac{1}{5}$ of the three bars. He could have taken $\frac{1}{5}$ of the three bars in a number of other ways.

(b) The diagram represents the ten records from three albums.



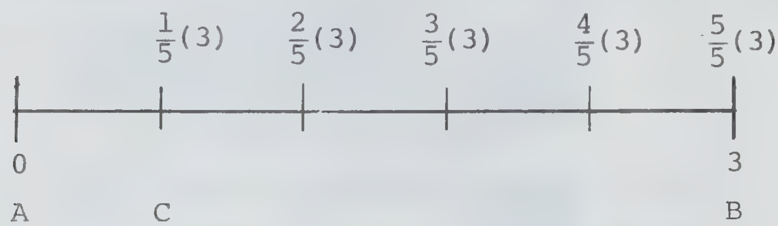
Leon chooses two records from the albums (the shaded ones). He has $\frac{1}{5}$ of the three albums.

(c) The length of segment AB is 3

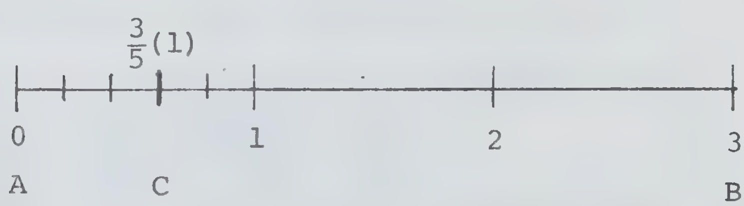


Locate C on AB so that AC = $\frac{3}{5}$.

Think of AC as $\frac{1}{5}$ of 3. Thus, divide AB into five equal segments, as shown below.



<u>Note</u>: This problem could also have been done by the method of i)(c) on page 9. Think of AC as $\frac{3}{5}$ of 1. Divide AB into three equal parts, and the first of these into five equal parts.

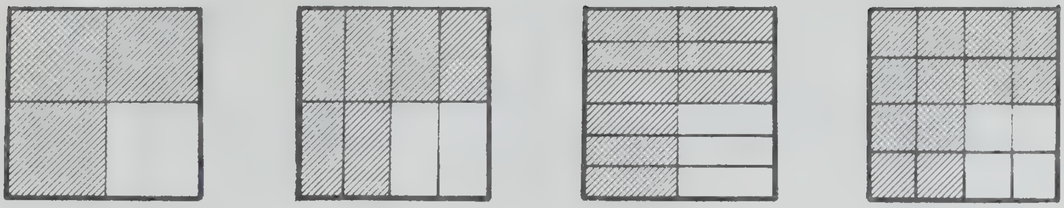


The above examples show some of the many situations in which fractions are used and illustrate the subtle shades of interpretation that it is possible to give to the symbol $\frac{3}{5}$. In summary, $\frac{3}{5}$ is used as an abbreviation for $\frac{3}{5} \times 1$, or $3 \times \frac{1}{5}$, or $\frac{1}{5} \times 3$. To interpret $\frac{3}{5}$ in a given situation, the student should determine which of the expanded forms is most meaningful to her or him.

<u>Equivalent Fractions</u>

There are numerous examples at the concrete and semi-concrete levels that justify the equivalence of two fractions. Some of these should be investigated as an introduction to the numeric rules for reducing fractions and for adding and subtracting fractions that have unlike denominators.

a) Part of the whole



$\frac{3}{4}$ of the square is shaded

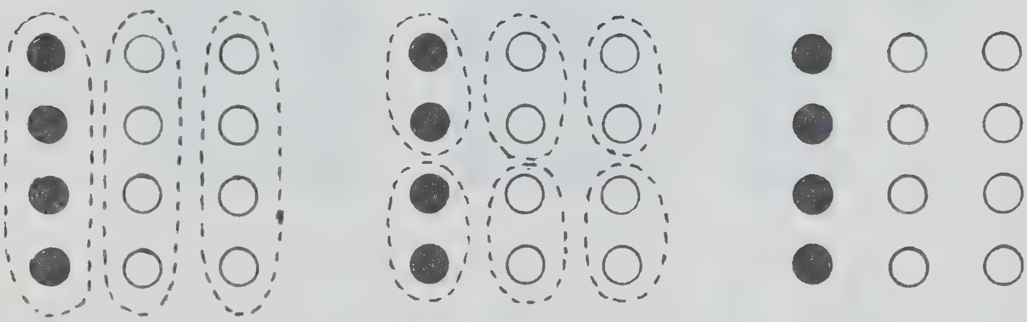
$\frac{6}{8}$ of the square is shaded

$\frac{9}{12}$ of the square is shaded

$\frac{12}{16}$ of the square is shaded

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$$

b) Part of a set of things



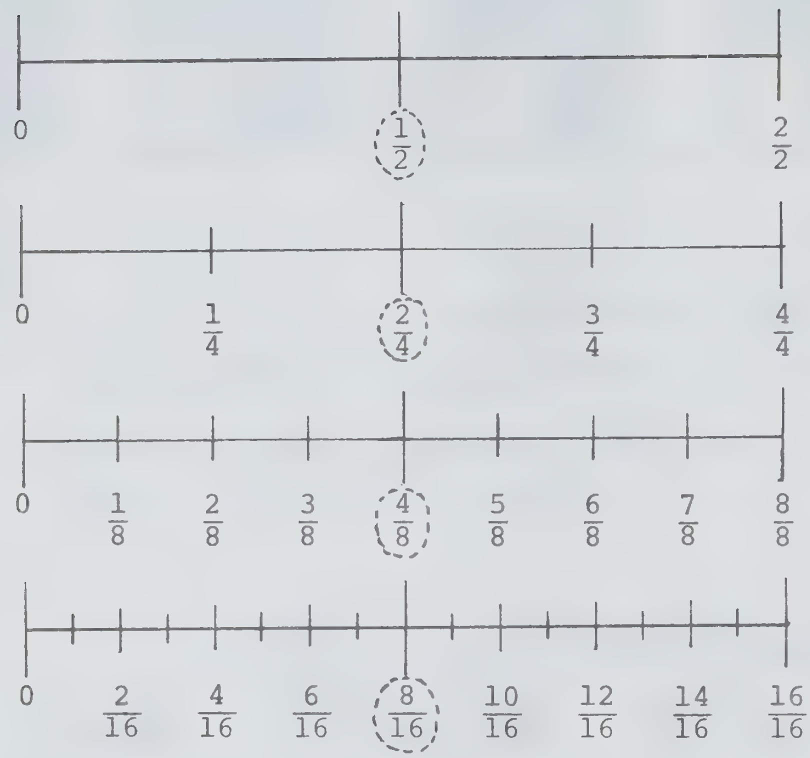
$\frac{1}{3}$ of the circles are black

$\frac{2}{6}$ of the circles are black

$\frac{4}{12}$ of the circles are black

$$\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$$

c) Part of a segment



$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}, \text{ etc.}$$

d) See examples ii)(b) on page 10, iii)(a)(b) on page 11. These can be used to illustrate equivalent fractions.

Mixed Numbers

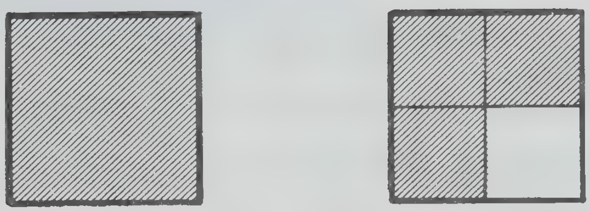
Expressions such as $1\frac{3}{4}$ and $2\frac{1}{2}$ represent mixed numbers; that is, numbers composed of a whole number and a fraction:

$1\frac{3}{4} = 1 + \frac{3}{4}$; $2\frac{1}{2} = 2 + \frac{1}{2}$

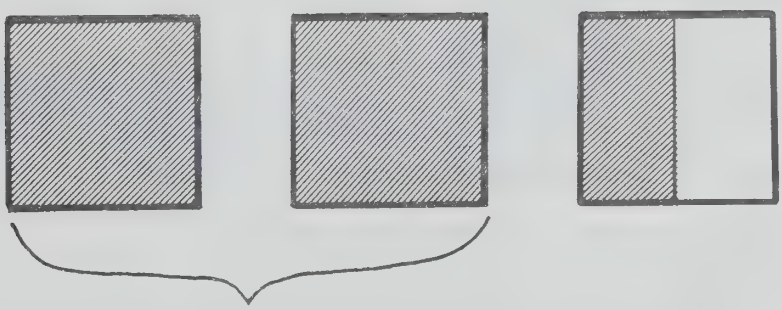
Mixed numbers may also be represented by decimals; for example

$1\frac{3}{4} = 1.75;$ $\qquad$ $2\frac{1}{2} = 2.5$

The meaning of mixed numbers can be illustrated by diagrams. For example,

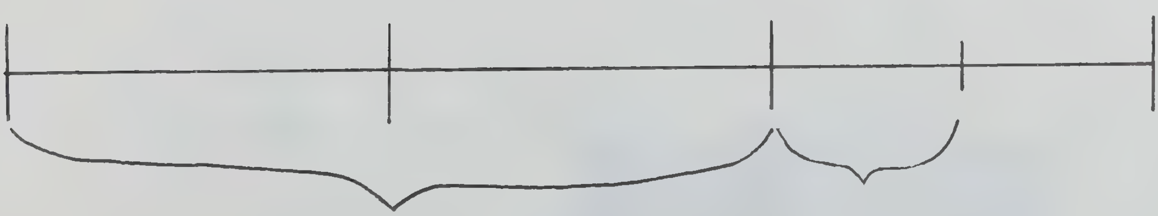


$1 + \frac{3}{4} = 1\frac{3}{4}$



$2 + \frac{1}{2} = 2\frac{1}{2}$

$1 + \frac{3}{4} = 1\frac{3}{4}$



$2 + \frac{1}{2} = 2\frac{1}{2}$

By dividing the 'wholes' into the appropriate number of parts, the above diagrams can be used to illustrate that

$$1\tfrac{3}{4} = \tfrac{7}{4} \qquad \text{and} \qquad 2\tfrac{1}{2} = \tfrac{5}{2}$$

b) Operations with fractions, reciprocals; practice; applications to real-life problems

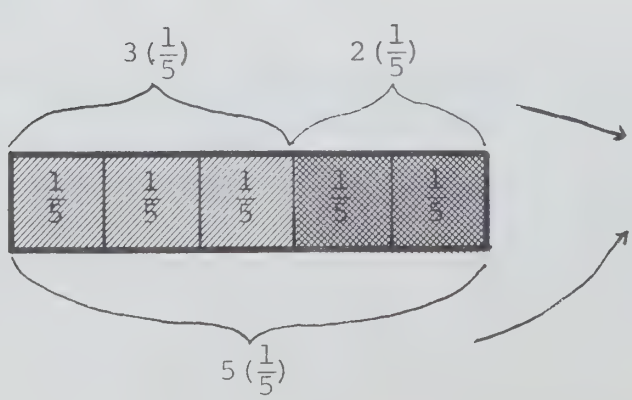
Operations with fractions have been introduced in the Junior Division. They should be reviewed with real situations and diagrams to consolidate the student's understanding of the concepts and skills.

Addition of Fractions

The addition of fractions depends on the concept introduced on page 10 in which a fraction such as $\frac{3}{5}$ can be considered as an abbreviation for $3(\frac{1}{5})$ or $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$.

The principles for adding fractions may be developed by stages as illustrated below.

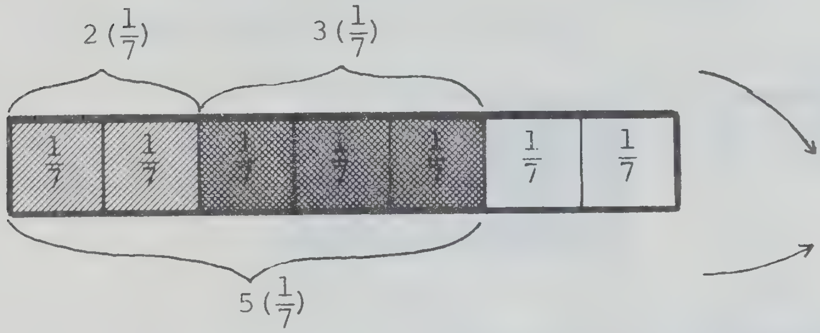
i) The fractions have the same denominators, and their sum is 1



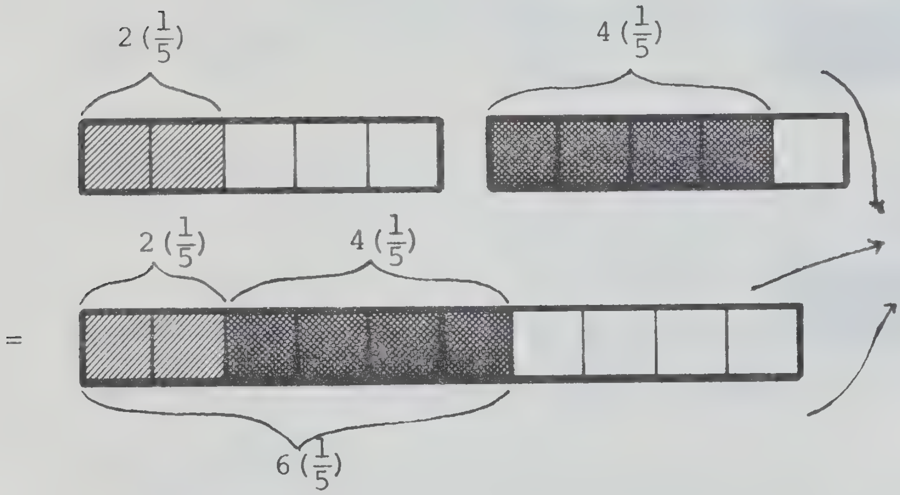
$$\begin{aligned} &\tfrac{3}{5} + \tfrac{2}{5} \\ &= 3(\tfrac{1}{5}) + 2(\tfrac{1}{5}) \\ &= 5(\tfrac{1}{5}) \\ &= \tfrac{5}{5} \\ &= 1 \end{aligned}$$

This can be illustrated with sets and a number line.

ii) <u>The fractions have the same denominators, and their sum is less than or greater than 1</u>

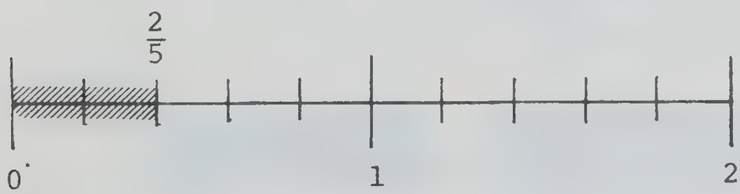


$$\frac{2}{7} + \frac{3}{7}$$
$$= 2\left(\frac{1}{7}\right) + 3\left(\frac{1}{7}\right)$$
$$= 5\left(\frac{1}{7}\right)$$
$$= \frac{5}{7}$$

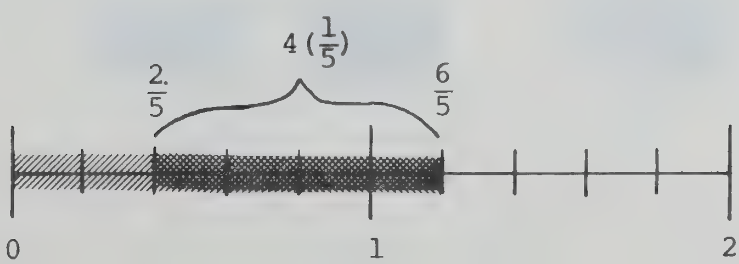


$$\frac{2}{5} + \frac{4}{5}$$
$$= 2\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right)$$
$$= 6\left(\frac{1}{5}\right)$$
$$= \frac{6}{5}$$

The same example can be illustrated on a number line. Divide each unit into fifths (as shown).

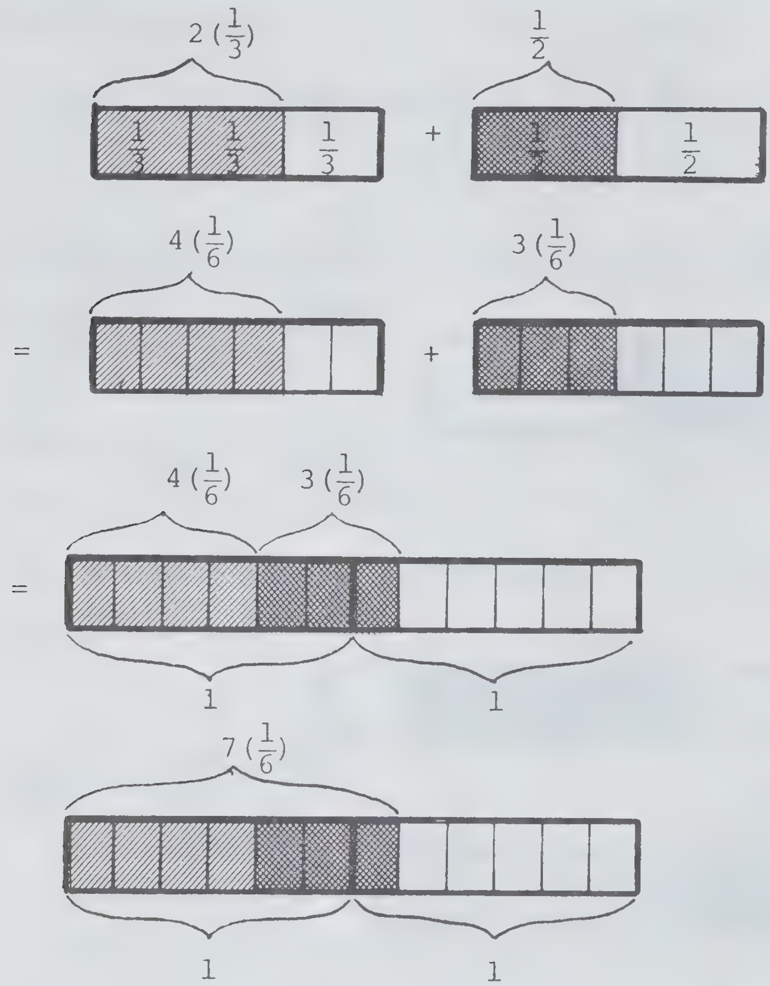


Locate $\frac{2}{5}$ on the line. Count to the right $4\left(\frac{1}{5}\right)$. This locates the point $\frac{6}{5}$.



$$\frac{2}{5} + \frac{4}{5}$$
$$= 2\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right)$$
$$= 6\left(\frac{1}{5}\right)$$
$$= \frac{6}{5}$$

iii) <u>The fractions have different denominators</u>



$\frac{2}{3} + \frac{1}{2}$

$= \frac{4}{6} + \frac{3}{6}$

$= 4(\frac{1}{6}) + 3(\frac{1}{6})$

$= 7(\frac{1}{6})$

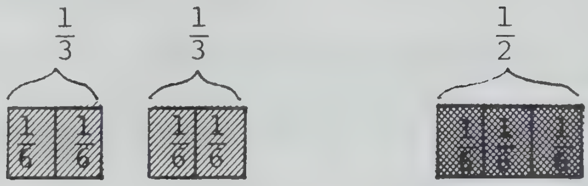
$= \frac{7}{6}$

The above example could be developed by using rectangular strips of paper cut from grid paper. For example,

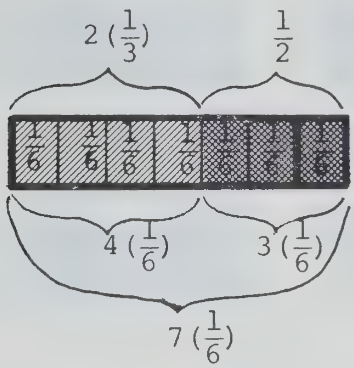
i) Make two strips 1 x 6.

ii) Cut one of these into thirds, the other into halves.

iii) Select $2(\frac{1}{3})$ and $1(\frac{1}{2})$.



iv) Place these end to end.



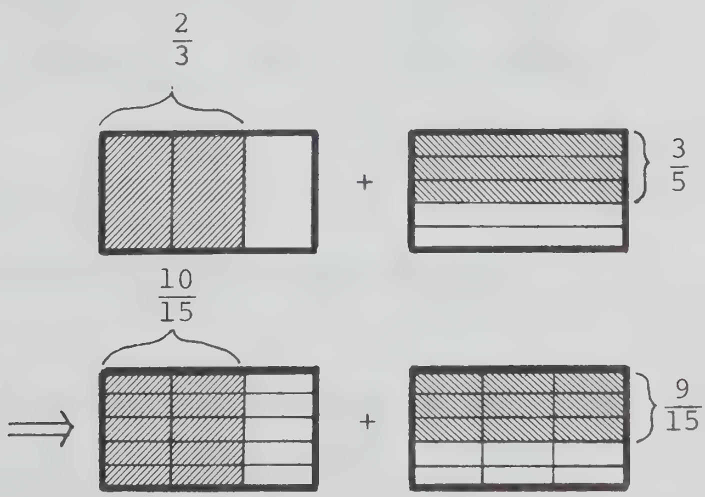
v) The sum $\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6}$

Establishing the principle of re-writing the fractions with the same denominator will necessitate practice with simple examples, such as:

i) $\frac{2}{3} + \frac{3}{5} = \frac{2}{3} \times \frac{5}{5} + \frac{3}{5} \times \frac{3}{3}$

$= \frac{10}{15} + \frac{9}{15}$

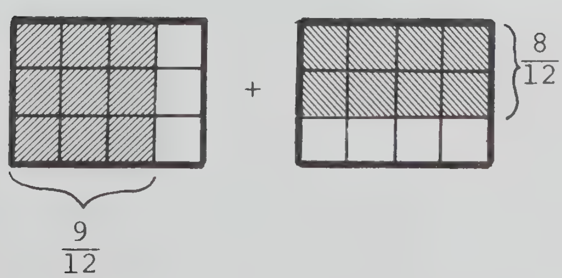
$= \frac{19}{15}$



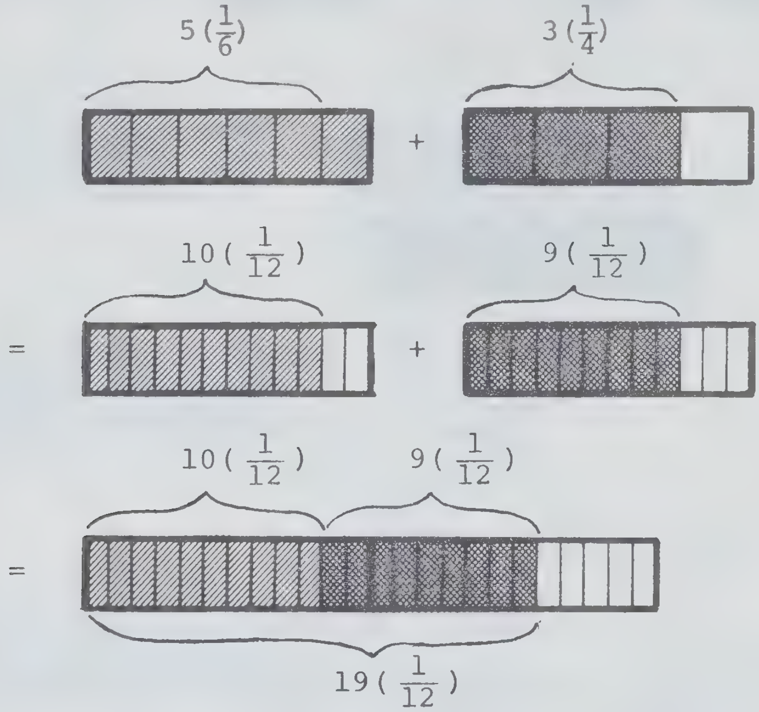
ii) $\frac{3}{4} + \frac{2}{3} = \frac{3}{4} \times \frac{3}{3} + \frac{2}{3} \times \frac{4}{4}$

$= \frac{9}{12} + \frac{8}{12}$

$= \frac{17}{12}$



The process for obtaining the <u>lowest common denominator</u> is discussed in the notes for 7N 5a. Simple examples such as $\frac{1}{2} + \frac{1}{4}$, $\frac{3}{8} + \frac{1}{4}$, and $\frac{5}{6} + \frac{3}{4}$ permit the procedure to be represented by diagrams on grid paper, if necessary. For example:



The search for the lowest common denominator need not be emphasized at this time since the pattern established by an example such as

$$\frac{3}{5} + \frac{7}{15} = \frac{3}{5} \times \frac{15}{15} + \frac{7}{15} \times \frac{5}{5} = \frac{3 \times 15 + 7 \times 5}{5 \times 15}$$

parallels the general algebraic procedure for adding fractions. This practice will be developed in a later grade as:

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \times \frac{d}{d} + \frac{c}{d} \times \frac{b}{b} = \frac{ad + bc}{bd}$$

Calculator Method

The algebraic rule becomes more significant and the lowest common denominator less so, when a calculator is used. For example, $\frac{3}{7} + \frac{5}{56}$ can be found with a calculator as follows.

$\boxed{3}$ $\boxed{x}$ $\boxed{5}$ $\boxed{6}$ $\boxed{M+}$ $\boxed{5}$ $\boxed{x}$ $\boxed{7}$ $\boxed{M+}$ $\boxed{M^R_C}$

gives the numerator 203.

$\boxed{7}$ $\boxed{x}$ $\boxed{5}$ $\boxed{6}$ $\boxed{=}$ gives the denominator 392.

By dividing 203 and 392 each by 7, $\frac{203}{392} = \frac{29}{56}$.

Alternately, $\boxed{M^R_C}$ $\boxed{\div}$ $\boxed{3}$ $\boxed{9}$ $\boxed{2}$ $= 0.5178571$,

if a decimal result is acceptable.

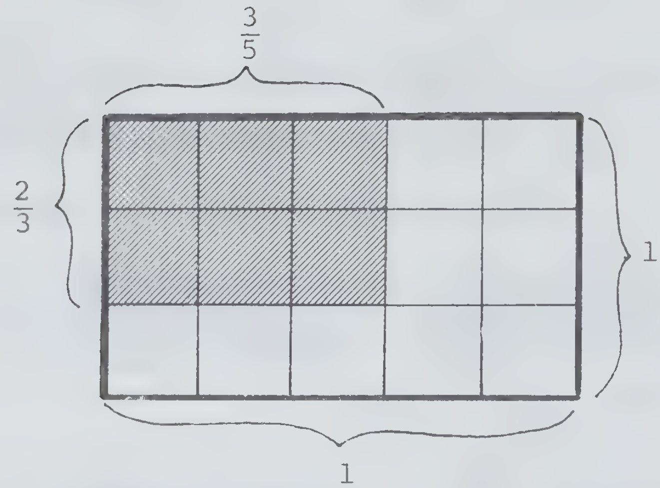
Of course, this decimal result could have been obtained directly by $\boxed{3}$ $\boxed{\div}$ $\boxed{7}$ $\boxed{M+}$ $\boxed{5}$ $\boxed{\div}$ $\boxed{5}$ $\boxed{6}$ $\boxed{M+}$ $\boxed{M^R_C}$

Subtraction with Fractions

Subtraction with fractions can be developed in the same way as addition of fractions.

Multiplication of Fractions

Multiplication of fractions may be demonstrated by a diagram, as follows for $\frac{2}{3} \times \frac{3}{5}$.



The unit square contains 15 regions (congruent rectangles); six of them are shaded.

The answer is $\frac{\text{number of shaded rectangles}}{\text{total number of rectangles}} = \frac{6}{15}$

Therefore $\frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$

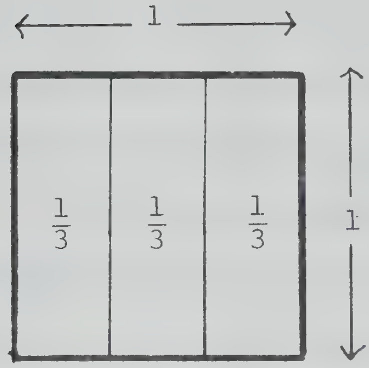
The rule may be generalized from a number of similar examples, as

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Division

Division with fractions may be introduced as follows.

i) The expression $1 \div \frac{1}{3}$ means "How many thirds in one?".
Consider a unit square. Divide it into thirds.

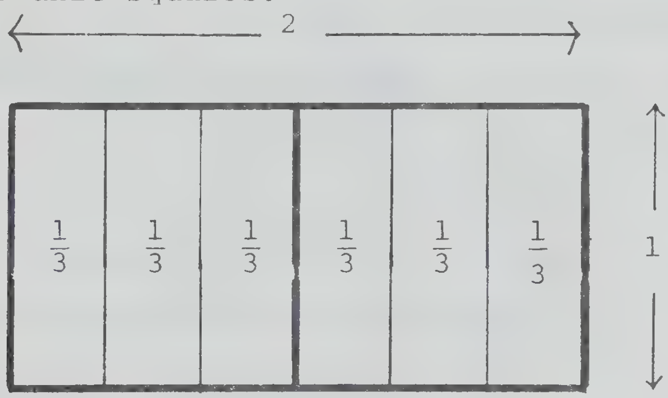


There are 3 thirds in 1.

Thus $1 \div \frac{1}{3} = 3$

ii) Similarly $2 \div \frac{1}{3}$ means "How many thirds in two?".

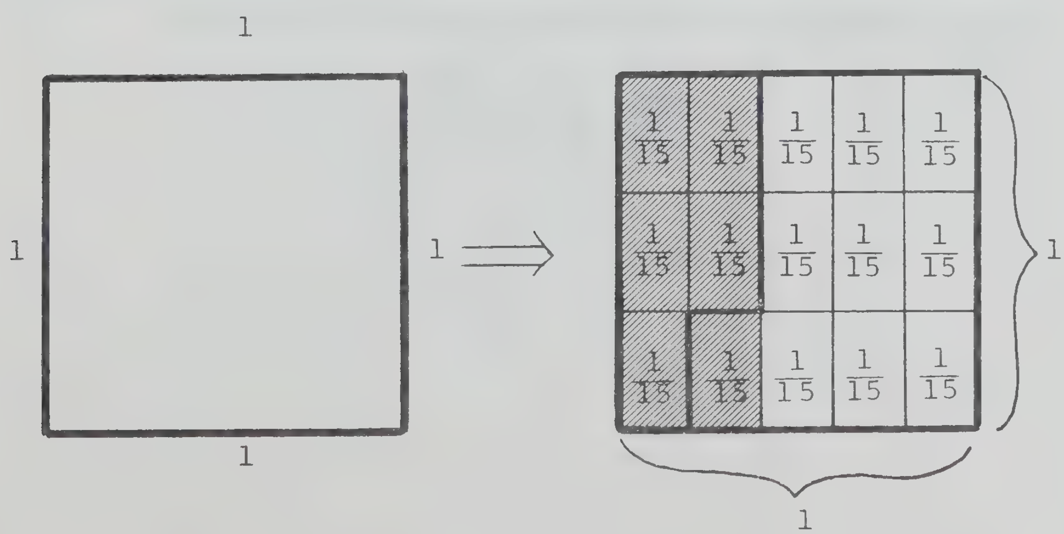
Use two unit squares.



There are 6 thirds in 2.

Thus $2 \div \frac{1}{3} = 6$

iii) Similarly, $\frac{2}{5} \div \frac{1}{3}$ means "How many one-thirds in two-fifths?".



The square is divided into 15 small rectangles.

One-third of the square (a row) equals five rectangles.

Two-fifths of the square (two columns) is shaded and equals six rectangles.

Therefore two-fifths of the square contains

five rectangles + one rectangle

i.e. (one-third of the square) + ($\frac{1}{5}$ of one-third of the square)

i.e. $\frac{6}{5}$ of one-third of the square

Numerically, $\frac{2}{5} \div \frac{1}{3} = \frac{6}{5}$

In each of examples i), ii), and iii), above, it may be observed that the answer can also be obtained by multiplying the dividend by the reciprocal of the divisor. For example,

$$1 \div \frac{1}{3} = 1 \times \frac{3}{1} = 3$$

$$2 \div \frac{1}{3} = 2 \times \frac{3}{1} = 6$$

$$\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \times \frac{3}{1} = \frac{6}{5}$$

This generalizes to the following rule:

A numeral divided by a fraction equals the numeral multiplied by the reciprocal of the fraction.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Practice

Students usually need considerable practice in the skills of reducing fractions to simplest terms, writing equivalent fractions, and performing operations with fractions. It is important, however, to know the procedures first and to be able to use them accurately in simple examples. In this way, they will experience success and will gain confidence with the techniques. There are many excellent games and activities that can be used to consolidate the students' skills and understanding of fractions.

Applications to Real-Life Problems

There will be fewer occasions in the metric future to use fractions in real-world situations. Ideally, practice and application should be extended over a period of time as appropriate examples surface.

Applications of fractions can be found in the following fields.

Music Whole notes, half-notes and quarter-notes provide opportunities to add and subtract with fractions. In $\frac{4}{4}$ time, how much time is left in a bar when ♩ ♩. is known?

<u>Cooking</u>	With time, recipes and measuring units will become more and more metric and there will be less need to use fractions here.
<u>Consumer Purchases</u>	Comparative shopping (5 for 77¢, what should be the cost of 1, 2, 3, and 4?) and unit pricing provide opportunities to practise with fractions and decimals.
<u>Ratio Problems</u>	This is discussed in the next topic.
<u>Sewing</u>	Sewing patterns are now officially metric; in practice the imperial system will phase itself out over a period of time.
<u>Measurement</u>	Length, area, volume, mass, temperature are officially measured with metric units and hence there are few opportunities to use fractions in a formal setting. Imperial measuring remains in some contexts, however, and will probably take some time to die out. These situations can be used for practice with fractions.
<u>Time</u>	Time is sometimes referred to by half-hours, quarter-hours. Problems in this context are usually solved by converting to minutes.

<u>Probability</u> Problems about probabilities involve work with simple fractions; the sum of all the probabilities for a given situation must be 1.

<u>Percentage</u> Percents are usually converted to decimals before computational work is done ($7\frac{1}{2}\% = 0.075$, etc.), and occasionally to fractions ($7\frac{1}{2}\% = \frac{7\frac{1}{2}}{100} = \frac{15}{200}$).

<u>Science</u> Many scientific principles are expressed in fractions, ratios, and proportions. Think of levers and pendulums, for example.

<u>Scale Drawings</u> The scale factor is often expressed as a fraction.

<u>Similar Figures</u> There are numerous real-world examples in which the lengths of sides (or areas, or volumes) of similar figures are compared. This is significant in practical trigonometry.

<u>Sports Records</u> Many sports records (e.g. for the high jump) have traditionally been recorded as mixed numbers. The use of metric measurement is increasing but many references are still made to older records expressed in imperial units and these often involve fractions.

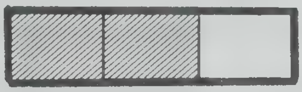
Industrial Arts — Activities that occur in Industrial Arts frequently involve fractions. Teachers should consult the industrial arts teacher to determine those activities that will continue to involve fractions rather than decimals in the future.

c) Ratios as comparisons

The notion of ratio is used in everyday situations and in mathematics to compare the sizes of two or more things. In some cases the ratio is implied. For example, "In a recent election, only three out of five of the people eligible to vote actually voted". The ratio idea could be made more explicit by saying, "The ratio of people who voted compared to those who were eligible to vote was 3 to 5". Ratios are useful in comparing the sizes of more than two things. For example, "The votes cast for the three candidates were in the ratio 5 to 4 to 2".

Notice that in the above examples, the notation 3:5 and 5:4:2 was not used; it seldom is in everyday writing. It is probably best to introduce the ideas of ratio to the students in an informal style, emphasizing the comparison of sizes in everyday language. The notation for a ratio can be introduced later. This is illustrated in the following examples.

i) In the rectangle below, the shaded part is twice the area of the unshaded part.

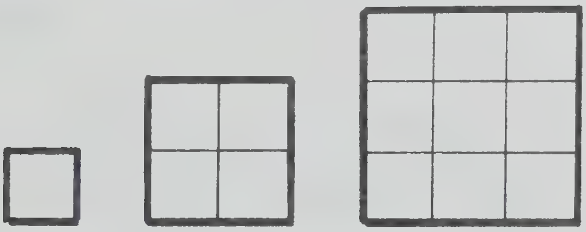


From this statement the following comparisons of sizes are implicit.

- The area of the shaded region compared to the area of the unshaded region is 2 to 1.
- The shaded region is $\frac{2}{3}$ of the whole rectangle, or the ratio of shaded region to the whole rectangle is 2 to 3.

Other comparisons can be made involving ratios of 1 to 2, 3 to 2, 1 to 3, and 3 to 1.

ii) For the three squares below,



the ratio of the lengths of the sides is 1 to 2 to 3, and the ratio of the areas is 1 to 4 to 9.

iii) In Ontario there is one medical doctor for every 577 people, and one nurse for every 153 people. The ratio of doctors to nurses is approximately 1 to 4.

It is important for students to realize that a ratio statement does not give the actual sizes of the things that are being compared. In examples i) and ii) above, the ratio statements were developed by comparing the actual sizes of the parts. However, given only the ratio statement we could not reconstruct these specific figures; there are many figures that would satisfy the statement.

To establish the point raised in the above paragraph, students might work from a given ratio to a table of possible values. This is illustrated in the example below.

Example Suppose your class is planning a party and decided to provide 5 hot dogs for every 2 people attending. The ratio of people to hot dogs is 2:5. The chart below gives a comparison of people to hot dogs.

People	Hot Dogs
2	5
4	10
6	15
8	20
.	.
.	.
.	.

By establishing tables such as the one above, the students can learn through practical experience that a ratio provides only the relative sizes of the things being compared. To know how many hot dogs to order, the number of students attending must

be known. Students should observe that each pair of numbers in the table is a specific case of the ratio 2 to 5.

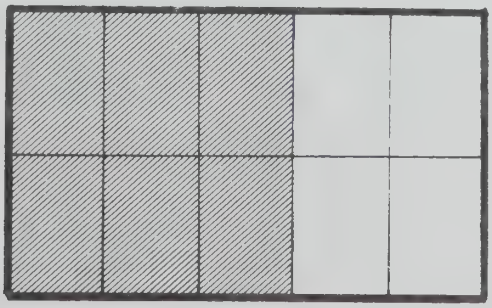
Ratios of Measured Quantities

When establishing a ratio to compare measured quantities (such as length, area, volume, mass, money, and speed), the measures must be in the same unit. For example, if the lengths of three segments are 15 mm, 2 cm, and 3 cm, then 15 mm can be changed to 1.5 cm, and the ratio written as 1.5 to 2 to 3, or 3 to 4 to 6.

Students should have practical experiences in establishing ratios for measured quantities. The Golden Ratio for a rectangle might be discussed at this stage; once more, this will illustrate that the ratio does not determine the specific lengths of the sides.

Ratios and Fractions

A ratio involves an ordered set of numbers, often notated as 2:3 or 2:5:7 or 2:0:5. In situations where a two-term ratio is used, frequently the fractional concept has meaning. This is illustrated by the following example.



Ratio Concept

Three out of five of the small rectangles are shaded; or more technically, the ratio of the shaded rectangles to all the rectangles is 3 to 5.

Further, the ratio of shaded rectangles to the white rectangles is 3 to 2.

Other ratio statements are possible.

Fraction Concept

$\frac{3}{5}$ of the large rectangle is shaded. (This implies the ratio statement that "the shaded rectangles compared to all the rectangles are in the ratio 3:5".)

The ratio concept is not the same as the fraction concept. A ratio uses two or more numbers to establish a comparison of the sizes of two or more things. A fraction represents only one number. However, two-term ratios (such as 3:2) are sometimes written as fractions ($\frac{3}{2}$) in many books. Teachers should recognize the potential confusion of this practice for students. Therefore, if the fractional notation is used for a ratio, the word ratio should accompany it in order to emphasize the comparison aspect rather than the single number aspect. Referring to the above diagram for example, write "the shaded part compared to the whole rectangle is in the ratio $\frac{3}{5}$".

d) Equivalent ratios; applications in practical problems

Equivalent Ratios

If the students have been introduced to ratio concepts through examples such as those given on pages 28 to 32, they will be aware that there are many pairs of numbers (triples, etc.) that represent the same ratio. Refer to the example on page 30; the ratio of people to hot dogs can be established as 2 to 5, 4 to 10, 6 to 15, and so on. These ratios are equivalent to each other. The term equivalent ratios should be introduced and the convention established that a ratio is in its simplest form when the terms are not divisible by a whole number other than 1. For example, the ratio 4:6:12 is not in its simplest form since 4, 6, and 12 are each divisible by 2. By dividing each term by 2, the equivalent ratio 2:3:6 is established. It is in the simplest form.

Proportions

A proportion is an equation relating two equivalent ratios. For example, 2:3 = 4:6 and 2:4 = 4:8 are proportions.

In many practical problems, three terms of a proportion are known and the fourth term is to be determined. The solution of such problems can be found by establishing a proportion of the form n:35 = 2:5. The student should realize that:

- 2:5 is the simplest form of the ratio;
- 35 = 5 x 7;
- then n = 2 x 7 or 14.

At this stage 14:35 should be tested to make sure it reduces to 2:5.

Applications in Practical Problems

There are many real-world situations in which the ratio concept is used. Some of these are listed below.

Sports	number of hits compared to number of times at bat; number of goals compared to number of shots on the net; ratio of games won to games played
Statistical Data	responses on surveys; advertising claims; betting odds; insurance data
Mechanical Devices	gear ratios; levers; 5- and 10- speed bicycles; pulleys
Geometric Properties	sides of right-angled isosceles triangle; shadows; similar figures; circumference to diameter
Scale Diagrams	maps; floor plans; designs of equipment
Science	scientific principles (proportions)

e) Rates, developed from real-world examples

There are many real-world situations in which two quantities are compared in different units; such comparisons are called rates. Examples of rates include:

- number of kilometres per litre (9 km/L);
- number of litres per 100 kilometres (gasoline consumption, 12 L/100 km);
- number of kilometres per hour (speed, 80 km/h);
- cost of an item per unit of measure (unit pricing, $2.50/kg);
- number of words typed per minute (85 words/min);
- time to complete a race over a specified distance (10.15 s/100 m);
- rate at which interest is paid on a sum of money (9.5¢/$1);
- number of heart beats per minute (75 beats/min);
- number of photocopies per minute (90 copies/min);
- pay in dollars per hour ($4.83/h);
- cost per half-ounce (postage rate for letters, 14¢/½ oz.; this rate will likely be metric in the near future);
- number of revolutions per minute; (example, records on a turntable — $33\frac{1}{3}$ r/min);
- cost per kilowatt hour (electrical rate, 7.00¢/kW•h; in the future this may change to cost per megajoule);
- number of beats per minute (metronome, music tempo, 70 beats/min);
- number of births (deaths) per year (561 births/a);
- mass of an object per unit of volume (2.3 kg/L);
- additional cost per word (beyond the first ten) for a telegram ($0.14/word);
- cost per agate line for an advertisement ($1.35/agate line);
- cost per additional minute for a long distance phone call.

Ratio and Rate

A ratio is a comparison of the sizes of two (or more) quantities in which the measures involve the same units. For example, two containers hold 1.5 L and 1 L of liquid; their capacities are in the ratio 3:2. The unit, litre, is not needed in the ratio, and is not included. In mathematics, a ratio involves only numbers.

A rate is a comparison of the sizes of two (and only two) quantities in which the measures involve different units. For example, a car travels 100 km on 8 L of gasoline; the gasoline consumption is 8 L/100 km. The units are essential to the meaning of the rate and must be included.

Notation

Rates are often written in fractional form, for example, as 8 L/100 km or as $\frac{8\text{ L}}{100\text{ km}}$; the slash (/) or fraction bar (——) is read as "per". Like ratio, a rate may be given in a number of equivalent forms. In a supermarket, sirloin steak may be advertised at $2.98/lb. (This will likely be $/kg in the near future.) Different packages of steak are priced at $5.21 for 1.75 lb., $6.85 for 2.30 lb., $3.69 for 1.25 lb., and so on. These are rates: $5.21/1.75 lb., $6.85/2.30 lb., and $3.69/1.25 lb. To check the pricing, these rates should be reduced to the cost per pound.

In most situations, the rate is considered to be in its simplest form when the second term is a single unit. For example, a nurse may count a person's heartbeat for 30 seconds as 37 beats/30 s. This would be converted to 74 beats/min. A person might type 840 words in ten minutes; this would be reduced to 84 words/min.

The following are examples of problems involving rate.

i) Anne is planning to go on a diet, and hopes to lose 5 kg in 30 d. How much should she lose per day?

Solution Rate of loss of mass = 5 kg/30 d

= 1 kg/6 d

(= $\frac{1}{6}$ kg/d)

This answer is more meaningful as 1 kg/6 d. Most scales will not be precise enough to record $\frac{1}{6}$ kg/d.

ii) Spiderman is planning to climb up the CN Tower. He estimates it will take two hours to reach the top. The tower is 553.3 m tall. At what average rate should he climb?

Solution Average Rate $= \frac{553.3 \text{ m}}{2 \text{ h}}$

$= \frac{553.3 \text{ m}}{2 \times 60 \text{ min}}$

$\cong 4.6$ m/min

Again, this is an approximate rate. Spiderman has guessed that he can climb the tower in two hours. His guess could just as easily have been two and a half hours. A reasonable answer to this whimsical problem would be 5 m/min. Note, he does not climb at a constant rate; he will

stop to rest at many stages and, when actually moving, will move at different rates.

iii) Bill reads about 25 pages of his library book a day. The book has 370 pages. How long will it take him to read the book?

<u>Solution 1</u> He reads 25 pages in 1 day,

or 250 pages in 10 days,

or 375 pages in 15 days.

It will take him about two weeks to read the book.

<u>Solution 2</u> He reads 25 pages in one day.

He will read 370 pages in $\frac{370}{25}$ days

$= 14.8$ days.

He will read the book in about two weeks.

iv) The instructions on the label of a prescription are "2 capsules a day before retiring". The bottle contains 16 capsules. How long will the pills last? (Ans. 8 days)

v) A plane climbs about 8 m/s. How long will it take to climb from 500 m to 3200 m?

<u>Solution</u> Change in elevation is 2700 m.

The plane climbs 8 m in 1 s.

Therefore it will climb 2700 m in $\frac{2700}{8}$ s

$= 337.5$ s

$= \frac{337.5}{60}$ min

$\cong 5.6$ min

The plane will climb to 3200 m in about 5½ minutes.

Some students may wonder whether the time should be found by $\frac{2700}{8}$ s, or 2700 x 8 s or 8 ÷ 2700 s. Checking the reasonableness of these answers will show that the second and third answers (6 h, and 0.0029 s) are not reasonable. Now the rationale for dividing by 8 may be discussed. Suppose the plane climbed:

1 m/s, the time would be 2700 s;

2 m/s, the time would be $\frac{2700}{2}$ s (twice the speed, $\frac{1}{2}$ the time);

4 m/s, the time would be $\frac{2700}{4}$ s (4 times as fast, $\frac{1}{4}$ the time);

8 m/s, the time would be $\frac{2700}{8}$ s (8 times as fast, $\frac{1}{8}$ the time).

vi) Compare the value of the price of the 25 mL, 50 mL, 100 mL, and 150 mL sizes of a popular brand of toothpaste.

Solution 1 The student might compare the cost per millilitre of each size. Although this procedure gives an accurate basis for comparison of costs, in one sense it is unrealistic because toothpaste is not available by the millilitre. Also the numbers obtained would be quite small and, for most people, difficult to compare.

Solution 2 Use the 25 mL as the basic unit. Then compare the cost of the other sizes with the 25 mL tube as the basic unit. Suppose the costs are respectively 59¢, 79¢, $1.09, and $1.39.

The unit cost of each size is as follows.

Size	Total Cost	Approx. cost/25 mL
25 mL = 1 unit	0.59	60¢
50 mL = 2 units	0.89	45¢
100 mL = 4 units	1.09	27¢
150 mL = 6 units	1.39	23¢

General Comments

1. Students should be encouraged to check the reasonableness of the answers. For example, in iii), they should think of two weeks as 14 days, and 14 days at 25 pages per day is 350 pages. The answer is reasonable. This example illustrates that answers to everyday problems should be rounded loosely. It would be overly pedantic to leave the answer as 14.8 days, or to round it to 15 days.

2. Students should be encouraged to create their own problems related to ratio and rate. These could be written by students on ditto masters and distributed as a set of exercises to be investigated by the class.

3. There are numerous experiments that could be investigated by students in which the set of results would include specific cases of a rate. Discussion of these results could lead to the generalization of the rate in its simplest form.

4. At this grade level, discussion of significant digits and consistency between raw data and the number of significant digits in the results would not be appropriate. The students should be encouraged to give rough approximations for answers, based on the exact calculations. See examples i), ii), iii), v), and vi) above.

Ministry of Education Ontario

GRADE 8 NUMERICAL METHODS

SECTION 1: NUMBER APPLICATIONS

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 6, 11
	Ed PJ Div:	Pages 68 to 71 From Counting to Calculation (a resource document)
	Gr 7:	N 1; N 2; N 5; N 6; N 8; A 1c
PRESENT	Gr 8:	N 2; N 3; N 4; N 5; N 6; N 7; A 1; A 3; A 4; G 3b; G 4bc
FUTURE	Gr 9 Gen:	N 1; N 2; N 3; N 4; N 5; N 6; N 7; A 1; A 3; A 5; G 1d; G 2; G 4bc; G 5
	Gr 9 Adv:	N 1; N 2; N 3; N 4; N 5; N 6; A 1; A 2; A 3; A 5; G 1b; G 2; G 4b; G 5
	Gr 10 Gen:	N 1; N 2; N 3; N 4; N 5; N 6; N 7; N 8; A 1b; A 2; A 3; G 1b; G 2; G 3
	Gr 10 Adv:	N 1; N 2; N 3; N 4; A 1; A 2b; A 3; A 4; A 5; G 1; G 2; G 5

a) Practice with whole numbers and decimals; applications in real-life situations

This topic allows a variety of activities in which the basic skills of whole numbers and decimals are practised. Activities should be interesting to the students and related, as far as possible, to their personal experiences. It is recognized that the students will be at different stages of understanding, of ability to recognize possible applications, and of speed and accuracy with these skills.

Activities used to identify specific needs may be in the form of games and puzzles. On the basis of the results of these activities, some students may be given help, while those who do not need remedial work are engaged in alternate activities.

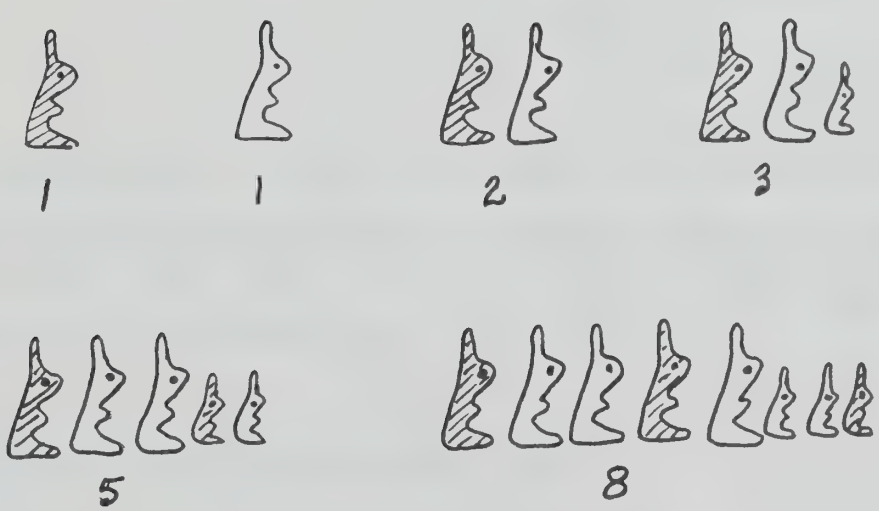
Routine drill exercises have a definite place in the strategy for consolidating skills and improving speed and accuracy. However, they should be used only with students who are deficient in these areas and then only after they understand the related algorithms.

Practice (games, puzzles, activities, remediation, drill, applications) should be used when needed throughout the year to maintain the skills at a functional level.

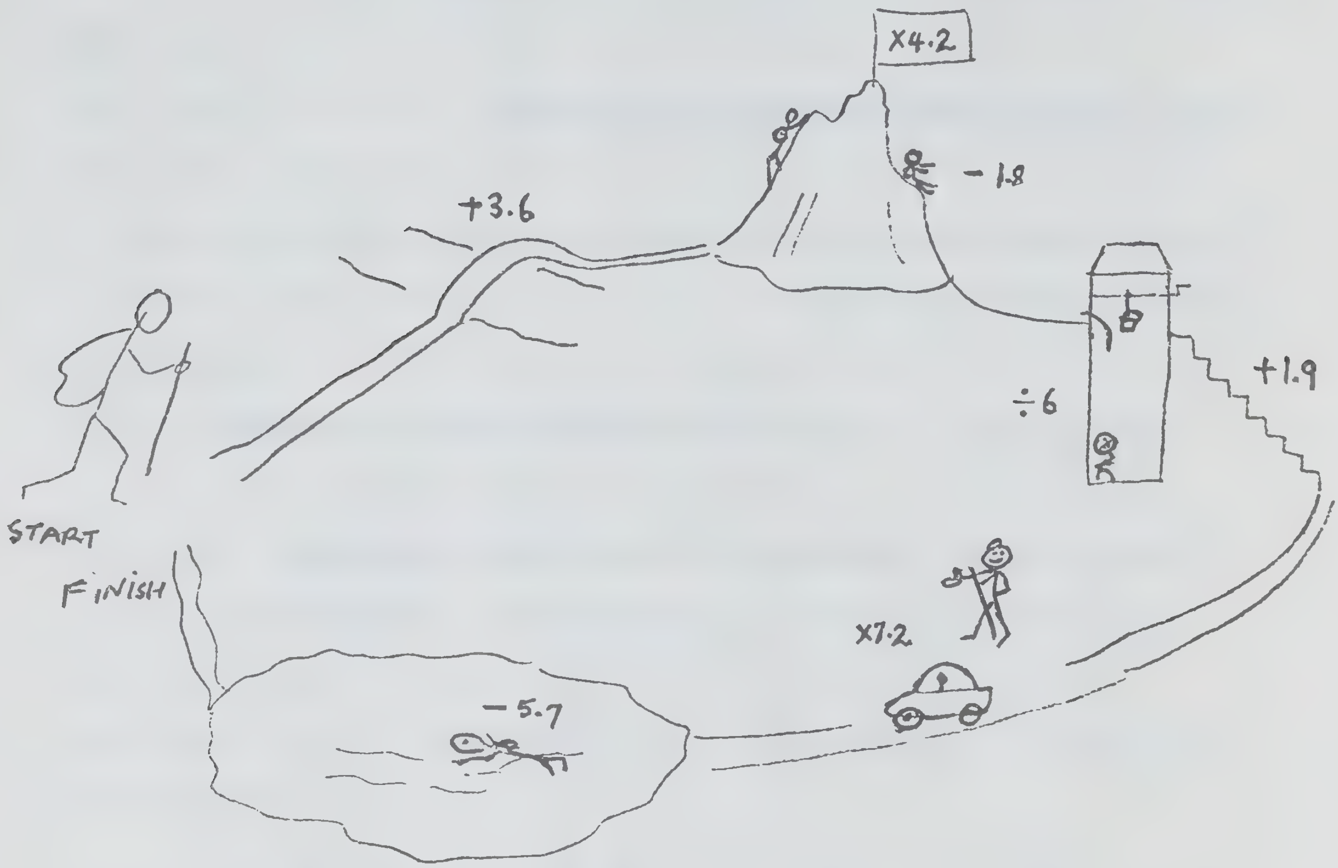
<u>Practice with whole numbers and decimals</u>

The following examples illustrate some activities that will provide useful practice.

1. Finding the terms in recursive sequences is popular with students. In particular, students enjoy the rabbit story associated with Fibonacci Sequence. This activity provides good practice in addition.

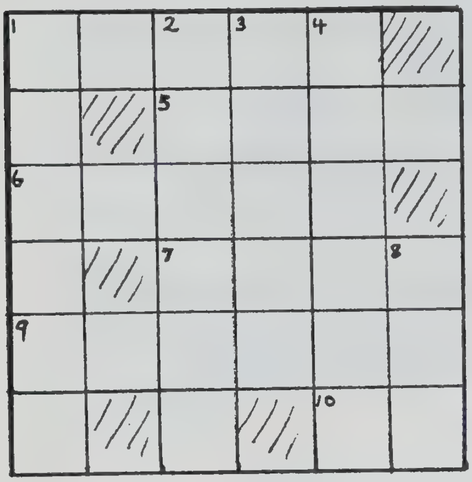


2. Flow Charts can be used quite effectively, especially if the setting is somewhat unusual.



Find the starting number which gives a final answer of 10. Because several trials are necessary, the division should be kept simple.

3. Cross-number puzzles are useful activities. The clues may contain decimals as well as whole numbers. The decimal point takes one square in the grid.



Across

1) π rounded to 4 digits
5) $1.02 + 0.61$
6) 27.32×3
7) $55.2 \div 3$
9) 8.546×0.6
10) 9^2

Down

1) $(3 \times 10^2) + (4 \times 7) + 0.53$
2) $\frac{100}{9}$ to 5 digits
3) $5 - 0.018$
4) $300 - [(11 \times 3) + (11 \times 0.02)]$
8) $20^2 + 8^2 - 2^2 + 1^2$

4. 'Grisley Grids' and 'Terrible Triangles' may be used; these are described in the notes for 7N 2.

Applications in real-life situations

The following examples illustrate ways in which these applications may be topical and timely.

1. In the spring, especially in rural and farming communities, it is possible to base calculations on seeding.
   - Estimate the number of seeds in a packet. For a statistical approach, pour the seeds on to a sheet of graph paper as uniformly as possible. Count the seeds in squares selected at random, then use ratio to find the number of seeds in the packet.
   - Use the directions on the packet with regard to plant and row spacings and work out the area needed for the mature plants.
   - Calculate the approximate value of the crop.

2. In the spring or early fall concentrate on outdoor activities.
   - How many times does a wheel on a bicycle turn in going from school to the swimming pool?
   - How long does it take to swim 10 km?
   - How long does it take to jog 10 km?

3. Finances
   - Estimate the annual income from a paper route for an assumed number of customers, and then calculate the amount.
   - Make a weekly balance of accounts.
   - Calculate the interest on the savings.

b) <u>Evaluation of powers with whole number exponents; applications</u>

Students require much practice with this particular mathematical notation. The meaning of the symbolism must be emphasized at this grade in order to avoid frustration later on. The words <u>power</u>, <u>base</u>, and <u>exponent</u> should be introduced in the proper contexts, as the students develop an understanding of this 'shorthand' way of writing particular numerals. Teachers should watch for students who use symbols casually, and 'drop' their exponents!

number of factors (exponent)

Power $\longrightarrow$ $2^3 = 2 \times 2 \times 2$

value of the factor (base)

Simple numbers should be used in order to focus attention on the meaning rather than on the calculations; for example:

$$\begin{aligned} 2^4 &= 2 \times 2 \times 2 \times 2 \\ &= 4 \times 4 \\ &= 16 \end{aligned}$$

$$\begin{aligned} 3^7 &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \\ &= 9 \times 9 \times 9 \times 3 \\ &= 81 \times 27 \\ &= 2187 \end{aligned}$$

Students should handle simple exercises such as:

$$\begin{aligned} 2^4 \times 5^2 &= 2 \times 2 \times 2 \times 2 \times 5 \times 5 \\ &= 4 \times 4 \times 25 \\ &= 16 \times 25 \\ &= 400 \end{aligned}$$

The real-life applications of powers are limited and are usually associated with physics and engineering formulae. Students may appreciate that the power concept is fundamental to an understanding of place value notation, i.e.

$$3728 = 3 \times 10^3 + 7 \times 10^2 + 2 \times 10^1 + 8 \times 1$$

In later grades, students will need to understand power notation when using the y^x key or the EE key on a calculator, and when using scientific notation.

c) Numerical development of the rules for multiplication and division with powers

Students should work with calculations involving numbers expressed as powers. The powers are expanded, the factors re-organized, and then the result re-expressed as a power.

Example

$$\begin{aligned} 2^3 \times 2^4 &= \underbrace{(2 \times 2 \times 2)}_{\text{3 factors of 2}} \times \underbrace{(2 \times 2 \times 2 \times 2)}_{\text{4 factors of 2}} \\ &= \underbrace{(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)}_{\text{7 factors of 2}} \\ &= 2^7 \end{aligned}$$

It may now be observed that the exponent 5 is the sum of the exponents 3 and 2.

When powers with the same base are multiplied, the intermediate step of adding exponents concentrates attention on the additive nature of the law.

For example:

$$5^3 \times 5^4 = 5^{(3+4)} \qquad \text{intermediate step; add the exponents}$$
$$= 5^7$$

Division with Powers

At the Grade 8 level, this topic deals only with powers with whole number exponents. Thus care should be taken that the numerator is greater than the denominator.

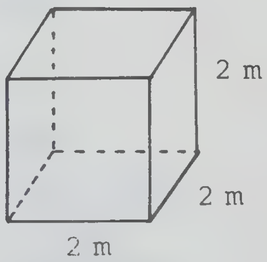
$$5^5 \div 5^2 = \frac{5 \times 5 \times 5 \times \overset{1}{\cancel{5}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \times \underset{1}{\cancel{5}}}$$
$$= 5 \times 5 \times 5$$
$$= 5^3$$

This may be summarized as:

$$5^5 \div 5^2 = 5^{(5-2)}$$
$$= 5^3$$

d) Geometric significance of the square and cube

Power notation also is used to represent dimensions in the physical world. Whereas 2^3 represents a number, $(2\text{ m})^3$ represents a physical entity such as a cube with edges of 2 m and volume 8 m^3, as shown.



Following their work in 7N 8 with volume and capacity, students should know that a cube with dimensions 2 m x 2 m x 2 m has a volume of 8 m^3, but that a 3-D figure with volume 8 m^3 is not necessarily a cube – it could be a cuboid, a sphere, a cylinder, or some other object.

Such discussions will help the students to think precisely and logically. For example, it is hoped that students will appreciate that $6 + (2 \times 8) = 22$ is a true statement, but that $6\text{ m} + (2\text{ m} \times 8\text{ m}) = 22\text{ m}$ is meaningless because linear and area measures cannot be combined.

e) Roots of perfect squares

There are many examples of everyday things we 'do' and then 'undo'; for example:

tie a shoelace	then	untie the lace;
take the cap off a pen	then	place the cap on the pen;
lock the door	then	unlock the door.

The same type of situation occurs frequently with mathematical operations; for example:

add five	then	subtract five;
multiply by two	then	divide by two;
subtract three	then	add three;
divide by four	then	multiply by four;

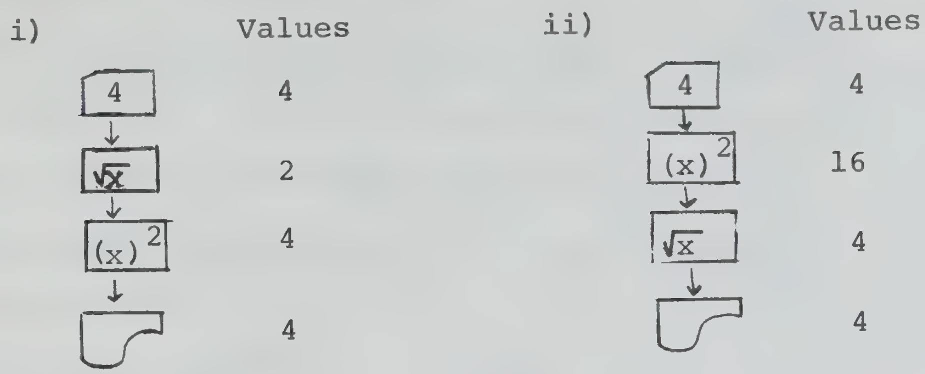
and so on.

Such pairs of operations are called inverse operations. Squaring a number and finding the square root (technically, the principal square root) of the result are inverse operations; for example:

$$5^2 = 25 \quad \text{and} \quad \sqrt{25} = 5$$

$$\sqrt{9} = 3 \quad \text{and} \quad (3)^2 = 9$$

This is illustrated by the following flow chart.



In each case the input value and the output value are identical; no change has occurred after the two operations. Students should have enough experience with squares of numbers to recognize the perfect squares 1, 4, 9, ..., 144, as well as squares such as 225, 400, and 625.

f) Using a calculator in the above topics; decimal factors

The purpose of this section, Number Applications, is a review, consolidation, application, and extension of the arithmetic skills associated with whole numbers, decimals, and powers and roots. Novel and interesting ways are recommended.

The calculator can be used in support of the above purpose. It will likely be highly motivating because it is a part of the world of mathematics that the students know outside the classroom. When used imaginatively, the calculator can complement the consolidation and extension of the skills of mental arithmetic or of pencil and paper arithmetic normally assumed at this grade level.

In beginning this work the students should have immediate recall of <u>at least</u> the following skills of mental arithmetic:

- given any number, the addition (or subtraction) of a number up to 12 to (from) it;
- multiplication facts up to 12 x 12;
- squares of numbers up to 12 and square roots of perfect squares up to 144;
- operations with powers that have whole number exponents.

Of course, many students will have skills well beyond this minimal list, but some may still need help. The aim is to bring students to the level where they can apply these skills readily in mathematical games, puzzles, and activities and in real-life applications.

The calculator may be used as an aid in:

- consolidating and extending the skills mentioned above;
- understanding the operations and applying them;
- investigating number patterns;
- dealing with applications in which there are many calculations, or in which the calculations would otherwise be tedious and would inhibit the flow of ideas;
- developing number sense through strategy games;
- sharpening comprehension of estimation, approximation, rounding, and truncation.

The examples that follow indicate some of the ways a calculator may be used for the above purposes.

Example 1 Addition Facts

Most mini-calculators have a constant function capability for each of the four arithmetic operations. For example, the machine can be 'programmed' to add the same number to any input value. Thus students, either individually or in pairs, may test their addition facts by comparing their own answers with those of the machine. The complexity of the exercises can be matched by the students to their individual capabilities.

i) Adding 8 (The constant operation is 'add 8'.)

Enter $\boxed{5}$ $\boxed{+}$ $\boxed{8}$

Think the answer 13 . Then test the result as follows.

Enter $\boxed{=}$; the display reads 13

The machine is now programmed to 'add 8' to any number that is entered. For example,

Enter $\boxed{1}$ $\boxed{6}$

Think 16 + 8 = 24

Enter $\boxed{=}$, the display reads 24

Vary the input

Enter $\boxed{2}$ $\boxed{3}$ $\boxed{4}$

Think 234 + 8 = 242

Enter $\boxed{=}$; The display reads 242

The student can now continue to refine the skill of adding 8 to any number of his/her choice.

The above process can be extended by replacing 8 by other numbers -- first between 2 and 12, then in the teens, then beyond.

ii) Adding Integers

In section N 4, integers may be used in the same way. However, different calculators may differ in methods of entering negative numbers; with some calculators the following exercise is not possible. If the machine has a $\boxed{+/-}$ button, try the following:

Enter $\boxed{7}$ $\boxed{+}$ $\boxed{6}$ $\boxed{+/-}$

Think 7 + (-6) = 1

Enter $\boxed{=}$; the display is 1

'Add -6' is now programmed into the machine.

Enter $\boxed{5}$ $\boxed{6}$ $\boxed{+/-}$

Think -56 + (-6) = -62

Enter $\boxed{=}$; the display is -62

Investigate the available machines to see if comparable techniques can be developed -- don't overlook the memory.

Example 2 Subtraction Facts

The process here is very similar to the process for addition.

i) Subtracting 9 (The constant operation is 'subtract 9'.)

Enter $\boxed{1}$ $\boxed{5}$ $\boxed{-}$ $\boxed{9}$

Think 15 - 9 = 6

Enter $\boxed{=}$; the display reads 6.

'Subtract 9' is now programmed into the machine.

Enter $\boxed{5}$ $\boxed{3}$

Think 53 - 9 = 44

Enter $\boxed{=}$; the display reads 44.

and so on ...

This process can be extended by subtracting 9 from larger numbers, by replacing 9 by other numbers from 2 to 12, then in the teens, then beyond.

ii) Subtracting with Integers

In section N 4, integers may be used in the same ways.

(a) Enter $\boxed{7}$ $\boxed{-}$ $\boxed{9}$ (The constant operation is 'subtract 9'.)

Think 7 - 9 = -2

Enter $\boxed{=}$; the display reads -2

Enter $\boxed{1}$ $\boxed{5}$ $\boxed{+/-}$

Think -15 - 9 = -24

Enter $\boxed{=}$; the display reads -24

(b) Enter $\boxed{7}$ $\boxed{-}$ $\boxed{9}$ $\boxed{+/-}$ (The constant operation is 'subtract negative 9'.)

Think 7 - (-9) = 16

Enter $\boxed{=}$; the display reads 16

Enter -56 (that is, $\boxed{5}$ $\boxed{6}$ $\boxed{+/-}$)

Think -56 - (-9) = -47

Enter $\boxed{=}$; the display reads -47

Example 3 Multiplication Facts

i) Multiplication by 6

Enter $\boxed{6}$ $\boxed{x}$ $\boxed{5}$ (The constant operation is 'multiply by 6'; see the note on the next page.)

Think 5 x 6 = 30

Enter $\boxed{=}$; the display is 30

Enter $\boxed{9}$

Think 9 x 6 = 54

Enter $\boxed{=}$; the display is 54

Enter $\boxed{4}$ $\boxed{5}$

Think 45 x 6 = 270

Enter $\boxed{=}$; the display is 270

Enter $\boxed{1}$ $\boxed{5}$ $\boxed{+/-}$

Think -15 x 6 = -90

Enter $\boxed{=}$; the display is -90

---

ii) Multiplication with Integers

Enter $\boxed{6}$ $\boxed{+/-}$ $\boxed{x}$ $\boxed{5}$ (The constant operation is 'multiply by negative 6'.)

Think 5 x (-6) = -30

Enter $\boxed{=}$; the display is -30

Enter $\boxed{5}$ $\boxed{+/-}$

Think -5 x (-6) = +30

Enter $\boxed{=}$; the display is 30

Note: the constant operation for multiplication is the first number entered; for addition, subtraction, and division it is the second number. (This is the way the constant operations (+, -, x, ÷) are built into the logic of most machines.)

Example 4 Division

i) Division by 6 (The constant operation is 'divide by 6'.)

Enter $\boxed{4}$ $\boxed{8}$ $\boxed{\div}$ $\boxed{6}$

Think 48 ÷ 6 = 8

Enter $\boxed{=}$; the display reads 8

Enter $\boxed{7}$ $\boxed{2}$

Think 72 ÷ 6 = 12

Enter $\boxed{=}$; the display reads 12

Enter $\boxed{5}$ $\boxed{6}$. The division is not exact.

Think 56 ÷ 6 > 9 and < 10

Enter $\boxed{=}$; the display reads 9.3333333

This constant function for division by 6 can be used to find numbers that are evenly divisible by 6.
It could be of value in developing recall facts about the '6 times' table. Also it encourages the student to find approximate results when the division is not exact.

This process can be extended so that 6 is replaced by other numbers from 2 to 12, then by selecting numbers greater than 12 (such as 15, 20, 25), then for other numbers in general with the numbers geared to the ability of the students.

Example 5   Addition and Subtraction as Inverse Operations

This topic should be investigated for particular pairs of operations such as: (add 15, subtract 15) or (subtract 39, add 39). For example:

Enter	Display
$\boxed{5}$ $\boxed{5}$	55
$\boxed{+}$	55
$\boxed{1}$ $\boxed{5}$	15
$\boxed{-}$	70
$\boxed{1}$ $\boxed{5}$	15
$\boxed{=}$	55

In the above exercise, 55 is the input number, add 15 is the first operation and acts on 55 to give 70; subtract 15 is the second operation and acts on 70 to give 55. Obviously the effect of add 15 is 'undone' by subtract 15. Each operation is called the inverse of the other.

When using large numerals, the numeral should be entered in the memory; for example:

Enter 39.2056 in the memory; that is enter

$\boxed{3}$ $\boxed{9}$ $\boxed{.}$ $\boxed{2}$ $\boxed{0}$ $\boxed{5}$ $\boxed{6}$ $\boxed{M+}$

Then enter $\boxed{5}$ $\boxed{7}$ $\boxed{2}$ $\boxed{+}$ $\boxed{M^R_C}$ $\boxed{-}$ $\boxed{M^R_C}$ $\boxed{=}$

The output is 572.

Example 6 Multiplication and Division as Inverse Operations

Use a process similar to the one above to show that multiplication and division by the same number are inverse operations of each other. For any numeral n, show that

$\boxed{n}$ $\boxed{x}$ $\boxed{9}$ $\boxed{\div}$ $\boxed{9}$ $\boxed{=}$ gives an output of n.

Note, however, that $\boxed{n}$ $\boxed{\div}$ $\boxed{9}$ $\boxed{x}$ $\boxed{9}$ $\boxed{=}$ may not give exactly n. For example, $\boxed{1}$ $\boxed{5}$ $\boxed{\div}$ $\boxed{9}$ $\boxed{x}$ $\boxed{9}$ $\boxed{=}$ gives 14.999999. The reason should be discussed; it involves truncation and approximation. By pencil and paper, $15 \div 9 = 1.6666666666...$ When the same division is done on the calculator, the result is truncated at the eighth digit (digits after the eighth are cut off) to give 1.6666666. If the calculator had rounded the last digit, then 1.6666667 would have appeared; and if this numeral is multiplied by 9 on the calculator, the result is 15. (The calculator would truncate 15.0000003 to 15.)

When doing long multiplication (or long division) with pencil and paper, the result can be tested by using the calculator to perform the inverse operation (rather than repeating the same operation on the calculator).

Example 7 Square and Square Root as Inverse Operations

If the calculator has $\boxed{\sqrt{x}}$ and $\boxed{(x)^2}$ buttons, then the inverse relation of these operations may be investigated. Initially this should be done with perfect squares. For example, investigate the following:

i) $\boxed{2}$ $\boxed{5}$ $\boxed{\sqrt{x}}$ $\boxed{(x)^2}$ (If the calculator does not have $\boxed{(x)^2}$, use $\boxed{x}$ $\boxed{=}$ in its place.)

ii) $\boxed{7}$ $\boxed{(x)^2}$ $\boxed{\sqrt{x}}$

This may be extended to examples such as the following:

iii) $\boxed{5}$ $\boxed{\sqrt{x}}$ $\boxed{(x)^2}$

The display reads 4.9999996. (This result may vary from calculator to calculator.) The output is approximately 5. Explain why the result is not exact.

---

iv) $\boxed{5}$ $\boxed{\sqrt{x}}$ $\boxed{\sqrt{x}}$ $\boxed{(x)^2}$ $\boxed{(x)^2}$ $\boxed{=}$

The display reads 4.9999987. Explain why the result is not exact.

---

v) Test the following:

$\boxed{5}$ $\boxed{\sqrt{x}}$ $\boxed{+}$ $\boxed{1}$ $\boxed{(x)^2}$ $\boxed{-}$ $\boxed{1}$ $\boxed{=}$

The display reads 9.472135, which is not 5 nor close to it.

The inverse operations must be adjacent.

Try $\boxed{5}$ $\boxed{\sqrt{x}}$ $\boxed{(x)^2}$ $\boxed{+}$ $\boxed{1}$ $\boxed{-}$ $\boxed{1}$ $\boxed{=}$

In this case the result is 4.9999996 which is approximately 5.

Note that the above examples help to reinforce the fact that the decimal representation of the square root of a non-perfect square (for example $\sqrt{5}$) is approximate. When the approximate root is squared, the result is approximate. For example $\sqrt{5} = 2.2360679$ (truncated after the eighth digit), and when this is squared the display reads 4.9999996.

The only completely accurate way of writing the square root of 5 is $\sqrt{5}$; $\sqrt{5} \times \sqrt{5} = 5$.

Example 8 Number Sense

A student can use a calculator to sharpen his or her number sense. For example, ask the students to change 15 into 243 by two operations of addition or subtraction. Suppose a student enters 15 then +240. The display then reads 255. The student would then need to select the operation that changes 255 into 243; i.e. -12.

The rules can be changed in the above example:

- to include three entries;
- to include only addition (in which the strategy is not to to go over, to get close but not over -- 'The Price is Right');
- to introduce a time element;
- to introduce decimals.

The above activity can be adapted to multiplication and division, or to multiplication only, or to division only.

The games become more complex when decimals are used. The student will learn to use the facts that:

- multiplication of the display by a number
  - i) larger than 1 increases the given number,
  - ii) between 0 and 1 decreases the given number;
- division of the display by a number
  - i) larger than 1 decreases the given number,
  - ii) between 0 and 1 increases the given number.

For example, convert 15 to 65 by using two operations of multiplication or division. If the student first multiplies by 4 the result is 60, then he or she must multiply the display by a number bigger than 1, but how much bigger? Students may compete with each other; the closest result after two moves wins. This activity is also applicable to decimals.

Example 9 Powers

The calculator may be used to investigate powers and to develop the rules for multiplication and division with powers (with whole number exponents). See the notes for 9Gen N 3e.

Example 10 Problem Solving

The calculator is useful for problems that need many calculations to determine the final answer. Without the calculator, many students resist a solution pattern that involves extensive calculations. Further, the main values of problem solving lie in establishing the method of solution, in finding the answer as efficiently as possible, and in testing the reasonableness of the result. The calculator makes it possible for the student to investigate a larger number of problems and so to have a wider variety of experiences.

The following is an example of a problem in which a calculator would facilitate the solution.

Refer to the seed problem on page 6. Suppose there are 100 squares, and that eight are chosen at random and have the following number of seeds: 17, 21, 28, 9, 16, 11, 10, 5. Now suppose that the plants must be 20 cm apart in each row, and the rows 55 cm apart. The value of each plant is 38¢. Find the size of the field needed for the crop, and the approximate value of the crop. This problem will take some time to work out by hand and furthermore the tedious paper work will deflect the student's attention from the important consideration: how to combine the numbers.

There are many references (sections of books, articles, work cards, ditto master sheets, manuals, paperbacks) that provide an almost endless collection of activities for the calculator. Although the calculator may be used to do routine computation and to check pencil and paper computation, it is recommended that it also be used:

- to develop and support arithmetic skills and conceptual ideas;
- to sharpen the students' ability with mental arithmetic, estimation, and approximation;
- to explore patterns;
- to improve the students' number sense;
- to expand the scope of problems to be solved.

Decimal Factors

When asked to express a number in factored form, we usually think of whole number factors; For example,

12 equals 2 x 6, or 2 x 2 x 3, or 3 x 4.

Sometimes it is useful to make use of fractional factors; for example: $\frac{5}{3} + 7 = \frac{1}{3}(5 + 21) = \frac{1}{3}(26) = \frac{26}{3}$.

In this case, 7 has been considered in factored form as $\frac{1}{3} \times 21$.

Numbers may also be expressed in decimal factored form. For example,

$$12 = 2.5 \times 4.8.$$

Thus

$$\begin{aligned} 2.5 \times 16.3 + 12 &= 2.5 \times 16.3 + 2.5 \times 4.8 \\ &= 2.5(16.3 + 4.8) \\ &= 2.5 \times 21.1 \quad \text{(factored form)} \\ &= 52.75 \end{aligned}$$

Although this process should not be stressed at this time, it has significant application in future work with matrices. At this grade level, it is sufficient for students to be aware that any number may be expressed as the product of many pairs of numbers when whole numbers, integers, fractions, or decimals are used as the factors. We simply choose factors that are useful in the problem being investigated. The calculator is a great aid in this activity.